MODELLING AND SIMULATION ESM 94

1994

EDITED BY
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and
DR. RAFAEL M. HUBER

UPC

SCS The Society for Computer Simulation International

SiE

JUNE 1- 3, 1994

UNIVERSITAT POLITECNICA DE CATALUNYA
BARCELONA
SPAIN

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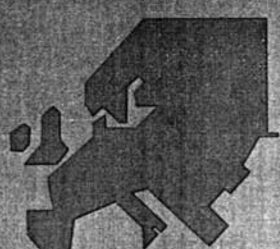

Proceedings of the Conference on Modelling and Simulation 1994
eds. Guasch and Huber

SIMULATION STUDY OF TECHNIQUES FOR THE SELF-ALIGNMENT OF A STRAPDOWN INERTIAL NAVIGATION SYSTEM

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Key words: navigation, alignment, simulation, modelling

ABSTRACT

Among the various techniques available to carry out the alignment process of a strapdown inertial navigation system, the most frequently used are gyrocompass and Kalman filtering. In this paper we study three of these methods: the classical gyrocompassing alignment, a low-order Kalman filter estimator and an open loop misalignment estimation procedure. Those methods are simulated and their numerical results are compared. Then conclusions about properties of the methods are derived.

LIST OF SYMBOLS

C_b^n Transformation matrix from body to navigational frames.
K_D, K_E, K_N Down, East and North loop gains.
L Latitude.
g Gravity.
$\varepsilon_D, \varepsilon_E, \varepsilon_N$ Down, East and North misalignments.
τ Time constant.
ω_a Accelerometers drift.
ω_g Gyros drift.
ω_{ie} Earth rotation
Ω Magnitude of Earth rotation.
ω_{cmd} Command rate.
t_s Settling time.
t_r Rise time.

Superscripts and subscripts

^{b} Body coordinates frame.
^{n} Navigational coordinate frame.
^{T} Transpose.
$_{D,E,N}$ Components in the Down, East and North directions.
$_{x,y,z}$ Components in the x, y and z directions.

INTRODUCTION

The alignment of a strapdown Inertial Navigation System (INS) is defined as the determination of the angular relationship between a vehicle-fixed set of axes and a reference or navigation coordinate frame. The initial alignment is a critical process since it has a bearing on the later implementation of the INS to determine the altitude as well as to compute velocity and position on the course of the navigation process. So, the performance of an INS can only be as good as the accuracy to which it is initially aligned.

Several techniques to accomplish alignment exist, including gyrocompassing and the use of Kalman filtering. Among those techniques, three methods of self-alignment are simulated and compared in this paper: The gyrocompass alignment, a low-order Kalman filter for the misalignment estimation and an open loop estimation procedure for the azimuth misalignment and the north gyro drift.

The three methods are applied to the alignment of an INS whose model has 23 states (Aranda et al. 1994).

The basic principle of gyrocompassing consists of feeding the appropriate level gyros and azimuth gyro with signals proportional to the accelerometers outputs or/and velocity error outputs (Britting 1971). Levelling is made first and then the azimuth is obtained by gyrocompassing.

The second method uses a Kalman filter for the misalignment estimation (Grewall et al. 1991, Aranda et al. 1994). However, a low-order Kalman filter yields practically the same accuracy as the full order Kalman filter. The low-order estimator has the following state vector: $(\varepsilon_N, \varepsilon_E, \varepsilon_D, B_N, B_D, x_D)$. Where $(\varepsilon_N, \varepsilon_E, \varepsilon_D)$ are the misalignment angles, (B_N, B_D) are the north and down component of the gyros biases and x_D is the down component of the accelerometers correlated noises.

Stieler and Winter (Stieler and Winter 1982) modifies the classical gyrocompassing procedure. They open the loop of the shunt integrator in the north-south channel and the loop for the vertical alignment. At the same time, the time constant of the levelling loop is reduced. A Kalman filter estimates the azimuth misalignment $\varepsilon_D \Omega \cos L$ and the north gyro drift component. The signals for levelling are used as measurement for the estimation procedure after

they have been integrated up and divided by the elapsed time.
In the three methods, the misalignment is modelled as a small-angle rotation, since an initial estimate of the transformation matrix is available. This initial estimate can be obtained by an analytic coarse alignment method.

In the following sections, we present the main aspects of the methods and simulation results with them, later on conclusions are derived.

GYROCOMPASS

The gyrocompassing loop of a stationary inertial platform is the electronic equivalent of a mechanical gyrocompass. In a Strapdown System, alignment means initialisation of the transformation matrix C_b^n for transforming the body-fixed measures into the navigational frame. It is the software equivalent of driving the misalignment angles to zero in a platform.

In this method, an estimation of the misalignment angles ε_N, ε_E is obtained from the measured acceleration vector with a low pass filter, and the command rate vector is generated by:

$$\vec{\omega}_{cmd} = \begin{bmatrix} -K_N & 0 \\ 0 & -K_E \\ 0 & -K_D \end{bmatrix} \begin{bmatrix} \varepsilon_N \\ \varepsilon_E \end{bmatrix}$$

This command rate is fed into the transformation matrix for integration.

An analog equivalent is:

$$\begin{bmatrix} \dot{\varepsilon}_N \\ \dot{\varepsilon}_E \\ \dot{\varepsilon}_D \\ \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\Omega\sin L & 0 & K_N & 0 \\ \Omega\sin L & 0 & \Omega\cos L & 0 & -K_E \\ 0 & -\Omega\cos L & 0 & 0 & -K_D \\ -g/\tau_1 & 0 & 0 & -1/\tau_1 & 0 \\ 0 & g/\tau_2 & 0 & 0 & -1/\tau_2 \end{bmatrix} \begin{bmatrix} \varepsilon_N \\ \varepsilon_E \\ \varepsilon_D \\ x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_{g_N} \\ \omega_{g_E} \\ \omega_{g_D} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1/\tau_1 \\ 1/\tau_2 & 0 \end{bmatrix} \begin{bmatrix} \omega_{a_N} \\ \omega_{a_E} \end{bmatrix}$$

Where x_1, x_2 are states for the low pass filters, and τ_1, τ_2 are the time constants of the corresponding states.

Neglecting the weak coupling between the levelling and gyrocompassing loops, the following two decoupled systems are obtained:

$$\begin{bmatrix} \dot{\varepsilon}_N \\ \dot{x}_1 \end{bmatrix} = \begin{bmatrix} 0 & K_N \\ -g/\tau_1 & -1/\tau_1 \end{bmatrix} \begin{bmatrix} \varepsilon_N \\ x_1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \omega_{g_N} + \begin{bmatrix} 0 \\ 1/\tau_1 \end{bmatrix} \omega_{a_E}$$

$$\begin{bmatrix} \dot{\varepsilon}_E \\ \dot{\varepsilon}_D \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \Omega\cos L & -K_E \\ -\Omega\cos L & 0 & -K_D \\ g/\tau_2 & 0 & -1/\tau_2 \end{bmatrix} \begin{bmatrix} \varepsilon_E \\ \varepsilon_D \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \omega_{g_E} \\ \omega_{g_D} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/\tau_2 \end{bmatrix} \omega_{a_N}$$

If the accelerometers uncertainties are constant then the north loop in steady state reduces to

$$\varepsilon_N(\infty) = \frac{1}{gK_N}\omega_{g_N} + \frac{1}{g}\omega_{a_E}$$

From here it is derived that no distinction can be made between the gyro drift North component and the accelerometer drift East component. The North drift gyro error can be reduced by shunting the K_N gain with an integrator (Stieler and Winter 1982).

The dynamic response corresponds to that of a second order system. The time constant is chosen to satisfy a signal settling time and a noise covariance. The K_N gain is chosen to provide adequate response rate and damping ratio.

In the gyrocompassing loop the dynamic of the angles ε_E and ε_D are given by the roots of the characteristic equation ($\tau=\tau_1=\tau_2$):

$$s\left(s^2 + \frac{s}{\tau} + \frac{K_E g}{\tau}\right) + s\Omega^2\cos^2 L + \left(\frac{K_D g\Omega\cos L}{\tau} + \frac{\Omega^2\cos^2 L}{\tau}\right) = 0 \qquad \text{(eq.1)}$$

If we choose K_D so that $K_D g/\tau << 1/\Omega\cos L$ then the characteristic equation roots are next to the roots of the equation $s\left(s^2 + \frac{s}{\tau} + \frac{K_E g}{\tau}\right)$. So, two conjugate complex roots are near the roots of

$$s^2 + \frac{s}{\tau} + \frac{K_E g}{\tau} = 0 \qquad \text{(eq. 2)}$$

and the real root is near s=0.

K_E and τ_2 are chosen on the condition that ε_E and ε_N evolve in similar way. Possible values are (Stieler and Winter 1982): $K_N=1/2g\tau$; $K_E=K_N$; $K_D=1/(16g\tau^2\Omega\cos L)$.

Table 1 shows the settling time (t_s) and the rise time (t_r) for both equations (eq. 1 and 2) with those gains.

Gyrocompassing becomes faster if levelling is made first, and the azimuth is assumed constant during this time. Figure 1 represents simulation results for a strapdown system with the values of sensor errors given in table 2. The chosen value for the time constant is τ=4 sec., during the levelling time. The time for this mode of operation is 60 sec. After that, the time constant is changed to τ=30 sec. for gyrocompassing. After 10 minutes, the errors are similar to the previous case.

	Eq. 2			Eq. 1		
	roots	t_s	t_r	roots	t_s	t_r
τ=1sc.	-0.5±j0.5	9.2	2.54	-0.5±j0.559 real -0.0473	9.2	2.4
τ=20sc.	-0.016667± j0.016667	276	76	-0.016667± j0.016736 real -1.6E-7	276	76
τ=120sc	-4.17E-3±j 4.17E-3	1104	305	-4.17E-3± j 4.17E-3	1104	305

Table 1: Roots of characteristic ecuation

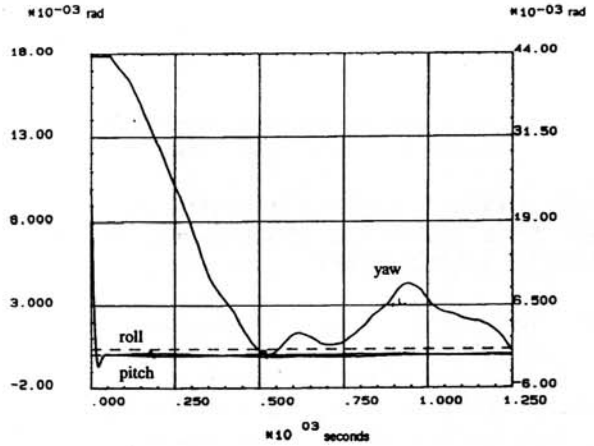

Figure 1: Roll, pitch and yaw errors. Gyrocompass alignment with initial coarse alignment

A LOW-ORDER KALMAN FILTER AS MISALIGNMENT ESTIMATOR

The alignment scheme employing the Kalman optimum techniques involves many state variables to describe the relative motion between the vehicle and the reference axes. This imposes heavy computation burden on the alignment process. In (Aranda et al. 1994), we studied a low-order Kalman filter, for the system we are considering, having almost the same accuracy as the full order one but with fewer state variables.

The analysis and design of Kalman low-order filters requires the following steps (Kortüm 1976): 1) Modelling of inertial sensor errors. 2) Obtaining of alignment equations with sensor error shaping filters. 3) Observability analysis. 4) Covariance analysis and error budget table. 5) From the covariance analysis, low-order filters are obtained. 6) Analysis of the proposed filter under parameter uncertainties. 7) Monte Carlo simulation.

1) Modelling of inertial sensor errors. Each Gyro is modelled by a random bias, a rate white noise (angle random walk), an angle quantization, a random ramp and a first order Markov process (pink rate noise), with measurement in angular increment (Aranda et al. 1990). Each accelerometer is modelled by white, quantization and correlated noises, with incremental velocity measurement. Table 2 shows these errors.

Coeficients:

Ω=15.041°/Hr=7.292E-5rad/sec; L=45°; g=9.84m/sec^2

Accelerometer correlated noise time constant = 300 sec.

Espectral density:

	White noise ($\sigma^2_{\omega_{rb}}$) m²/sec³	Correlated noise ($\sigma^2_{\omega_c}$) m²/sec⁴	Cuantification noise ($\sigma^2_{v_c}$) m²/sec²
Acc.1:	0.23E-6	0.23E-8	0.64E-8
Acc.2:	0.24E-6	0.24E-8	0.65E-8
Acc.3:	0.25E-6	0.25E-8	0.65E-8

	Rate white noise ($\sigma^2_{\omega_{rb}}$)	Cuantification noise (σ^2_r)
Gyro 1:	0.304E-11rad²/sec;	0.16E-10rad²
Gyro 2:	0.303E-11rad²/sec;	0.15E-10rad²
Gyro 3:	0.305E-11rad²/sec;	0.17E-10rad²

Initial uncertainty

Initial misalignment: $\sigma^2_{\varepsilon_N} = \sigma^2_{\varepsilon_E}$ = 0.9E-5rad²;

$\sigma^2_{\varepsilon_D}$ = 0.01rad²

Accelerometer bias: σ^2 = 3.5E-7 m²/sec⁴

Initial correlated noise: $\sigma^2_{acc.1}$ = 3.45E-7 m²/sec⁴;

$\sigma^2_{acc.2}$ = 3.6E-7 m²/sec⁴; $\sigma^2_{acc.3}$ = 3.75E-7 m²/sec⁴;

Gyro bias: σ^2= 0.6E-11 rad²/sec²

Table 2: Simulation parameters

2) For a strapdown system with the previous error models, the misalignment error equations with the shaping filters have a state vector with 23 components: three states for the misalignment angles (ε_N, ε_E, ε_D), one state to integrate the output of each sensor, three states to model each accelerometer, two states to model each gyro. However, this would not be economic, because some of the terms may be taken into account whose influence on the system behaviour is not significant, and some terms may not be observable and there may be, therefore, an estimator of lower order yielding the same accuracy as the full-order Kalman filter.

3) The observable and unobservable parts of the system can be separated, but the choice between them is not unique since the transformations are innumerable (Jiang and Lin 1992). The set of observable states are (Aranda et al. 1994): the misalignment angle ε_N with the east bias

accelerometers' components; the ε_E with north bias accelerometer component; the ε_D with east bias accelerometers and east drift gyros; the north and down bias accelerometers' components; the down accelerometers noise integral component; the accelerometers correlated noise; the gyro bias; the north and east acceleration integral, the rate integral with the integral of north, east and down gyro noise.

4) Covariance analysis and error budget table. Developing an error budget involves determining the individual effects of a single error source, or group of error sources (Gelb 1974). Table 3 shows the error budget. We can see that the accelerometer errors have more influence on the final estimate than the remaining sources of error.

	ε_N		ε_E		ε_D	
Noises	100sc.	300sc.	100sc.	300sc.	100sc.	300sc.
Gyro N	27%	19%	0%	0%	2%	6%
Gyro E	58%	57%	0%	1%	17%	50%
Gyro D	0%	0%	0%	0%	10%	16%
Acc. N	8%	9%	93%	98%	90%	87%
Acc. E	97%	99%	2%	0%	54%	83%

Table 3: Error budget

5) From the covariance analysis, low-order filters are obtained. After the observability and covariance analysis, we arrive at a model with the following state vector: (ε_N, ε_E, ε_D, B_N, B_D, x_D). Where (ε_N, ε_E, ε_D) are misalignment angles, (B_N, B_D) are north and down components of gyro bias and x_D is the down component of the accelerometers' correlated-noises.

6) Analysis of the proposed filter under parameter uncertainties. To find the sensitive of the filter with respect to some parameters' uncertainties, only those parameters are changed. The parameters changed are: C_b^n, the accelerometer correlation time and the variances of the noises of the sensors.

7) Finally, a Monte Carlo simulation is made (Aranda et al. 1994).

The alignment means updating the transformation matrix C_b^n by estimation of the misalignment error angles. The C_b^n updating is made by a first order approximation (Savage 1984):

$$\hat{C}_b^n(t_2) = \left(I - E^n(t_1)\right)\hat{C}_b^n(t_1)$$

Where I is the identity matrix and

$$E^n = \begin{bmatrix} 0 & -\varepsilon_D & \varepsilon_E \\ \varepsilon_D & 0 & -\varepsilon_N \\ -\varepsilon_E & \varepsilon_N & 0 \end{bmatrix}$$

That is equivalent to feeding the command rate into the matrix integration algorithm in the gyrocompass method.

Figure 2 shows the roll, pitch and yaw errors for a simulation. Levelling (updating ε_N and ε_E only) is made for 30 seconds, then the three angles are updated every 180 seconds.

A covariance analysis was performed with the following results: After 180 seconds the ε_D standard deviation is nearly to 8 mrad. Therefore, the azimuth errors are into these bounds. Furthermore, after 120 seconds the variance decreases slowly. Then, it needs more time to get less error.

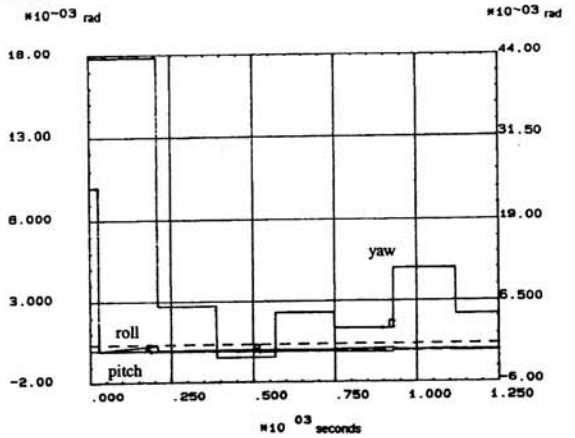

Figure 2: Roll, pitch and yaw errors. Alignment by a low Kalman filter as misalignment estimator

THE OPEN LOOP ESTIMATION PROCEDURE FOR AZIMUTH MISALIGNMENT AND NORTH GYRO DRIFT

The procedure shown here is proposed in Stieler and Winter 1982. The loops of the shunt integrator in the north-south channel and for the vertical alignment are opened. At the same time the time constant of the levelling loop is reduced. The torque signals for levelling the platform or the transformation matrix are used as measurements for the Kalman filter after they have been integrated up and divided by the elapsed time.

This measurement gives the estimation for the gyro drift north component and for $\varepsilon_D \Omega \cos L$. If measurements for the Kalman filter are taken in time increments greater than the longest period of the base motion, the modelling of the base motion as required in a Kalman filter may be neglected it can be assumed that each measurement is a signal for the north gyro drift D_N or the azimuth misalignment $\varepsilon_D \Omega \cos L$, contaminated by noise dependent of sensor and readout quality and base motion dynamics.

The characteristic equation can be show to be:

$$s\left(s^2 + \frac{s}{\tau} + \frac{K_E g}{\tau}\right) + \Omega^2 \cos^2 L\left(s + \frac{1}{\tau}\right) = 0$$

So that the roots of this equation are near s=o and of those of the following polynomial: $s^2 + \frac{s}{\tau} + \frac{K_E g}{\tau}$.

In the steady state and with constant drift signal, the fed signal is $(\varepsilon_D(0) + \omega_{g_D})\,\Omega\cos L$. The signal variance is reduced by averaging.

As in the levelling loop, after the settling time ($\approx 10\tau$) the measurement process and estimation of the states ω_{g_D} and $\varepsilon_D \Omega \cos L$ starts. Assuming that both loops are decoupled the filter is of first order.

The north gyro measurement is balanced from gyro north drift estimation, a C_b^n is updated by means of the estimation of ε_D.

Figure 3 shows the azimuth simulation error; the roll and pitch are as in figure 1. During 60 seconds the system is levelled, after that the azimuth is estimated and the matrix is updated each 60 seconds. In 4 minutes an error of 2.5 mrad is obtained.

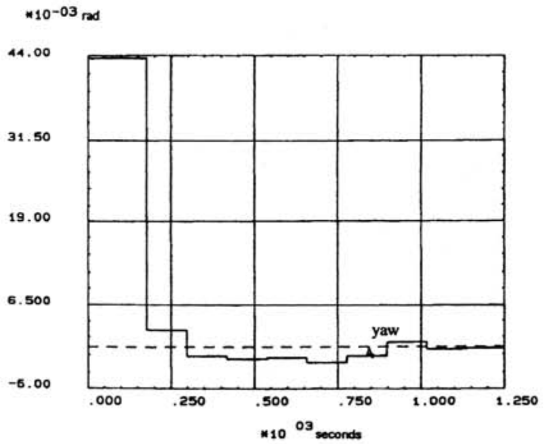

Figure 3: Yaw error. Open loop estimation procedure for azimuth misalignment

CONCLUSIONS

Three different methods for strapdown systems alignment have been studied. Those methods were simulated and from the numerical results we draw the following conclusions:

Gyrocompassing has the lest computer time and load of the three methods. However, the gains have to be chosen as a compromise between fast reaction, what requires high gains, and noise sensitivity, what requires lower gains. In that way the selection of the gains has to be made after a trial and error procedure.

The low-order Kalman filter has the higher computer time and load. It is sufficient to implement a five order Kalman filter after a correct tune of the noise levels to obtain an almost identical accuracy as the full-order 23-states Kalman filter considered.

The third method is a compromise between the other two methods. Now the levelling loops use constant gains as in the gyrocompassing but higher values of them are chosen so that the levelling phase is speedily-up. A Kalman filter of only two decouples states is used to estimate north drift and the azimuth misalignment. The Kalman filter parameters have to be chosen in a trial and error procedure. After an optimum choice of the parameters the accuracy of this method is quite similar to that of the low-order Kalman filter. That makes the method the most suite when a compromise accuracy and computation is required.

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