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# QFT Robust Control Design by Interactive Approach

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*Abstract*—In this paper an illustrative robust regulation problem is solved using a new QFT design interactive software tool. The proposed example is the same used in the manual of QFT Frequency Domain Control Design Toolbox to explain its use. Thus, a habitual user of this Matlab toolbox will be able to easily perceive the advantages and limitations of the new interactive software tool. This tool can be used for novice users in order to learn and understand the basic concepts involved in QFT design. However, it can be used for advanced users to solve real problems of robust control design.

## I. INTRODUCTION

THE Quantitative Feedback Theory (QFT) methodology for frequency domain design of robust controllers was created and developed by Isaac Horowitz [1]. It is a frequency domain design technique, which allows robust controllers to be obtained that fulfil some minimum features, formulated quantitatively, considering the existence of plant model uncertainties and perturbations. Full introductions to QFT with a large number of references can be found in [2]-[5].

There are currently many different software tools all aimed at helping the designer implement the different stages of QFT methodology. The most widespread and well known of all the existing tools is the QFT Frequency Domain Control Design Toolbox (FDCDT) [6]. These tools are typically implemented as a set of Matlab functions which the user can use to write his/her own scripts in accordance with the design problem that needs to be solved. On the other hand, the majority of these tools contain small graphic interactive programmes based on Matlab's Graphical User Interface which are more or less easy to use. However, the interaction of these programmes is reduced to either a simple interaction with a single graph, normally the Nichols diagram or filling out the fields in a dialog box in order to configure the values of certain parameters of the design.

A Quantitative Feedback Theory Interactive Tool (QFTIT) for single-input single-output systems has been developed to help users learn and understand the basic concepts involved in QFT methodology, and to provide them with a strong

theoretical base and engineering skills, which is a major issue in automatic control education [7]. QFTIT helps demystify abstract mathematical concepts involved in QFT design through visualization, while enhancing user skills, motivation, and the ability to understand and solve control problems [8].

QFTIT was developed using the interactive design CAD tool Sysquake, developed in the Institut d'Automátique of the Federal Polytechnic School of Lausanne [9]. Using Sysquake it is possible to develop tools that display several graphics simultaneously, some of which contain elements that can be manipulated with a mouse. The main differences between QFTIT and the others tool is are its ease of use and interactive nature. All that the end-user has to do is to place the mouse pointer over the different items that the tool displays on the screen. Any manipulation carried out on the screen are immediately updated and reflected on all the graphs generated and displayed by the tool. This allows users to visually perceive the effects of their actions during the controller design. The tool is freely available in the form of an executable file for Windows or Mac based platforms. The reader is cordially invited to visit the website http://ctb.dia.uned.es/asig/qftit/ to experience the interactive features of QFTIT.

The paper is organized as follows. In Section II we briefly describe the basic characteristics of QFTIT tool. Sections III to VII are devoted to explain how QFTIT solve the different stages in the QFT control design. These stages are templates computation, specifications, loop-shaping, pre-filter design and validation. An illustrative QFT control design example is solved using the tool in Section VIII. Finally, Section IX gives some concluding remarks and considerations about future work in order to improve the tool.

## II. BASIC FEATURES OF THE INTERACTIVE TOOL

For the purposes of QFTIT a block diagram of a Single-Input-Single-Output (SISO) feedback system loop with a controller having two degrees of freedom is shown in Figure 1.
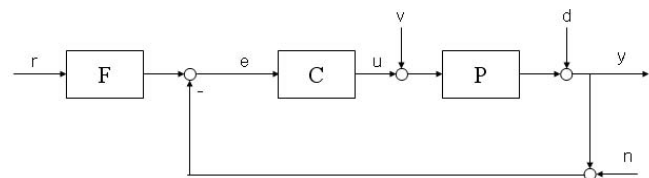


Fig. 1. Feedback system.

*F* is the transfer function of a pre-filter acting on the reference input *r*. *C* is the controller which, depending on an error signal *e,* generates a control signal *u* over the set of

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transfer functions of the plant $P$. This set describes the uncertainty region of the plant's parameters. $P$ may be subject to disturbances at its input $v$ and/or at its output $d$. The output signal $y$ may be affected by a measurement noise $n$.

Because the present version of the tool has only implemented the templates computation algorithm proposed in [10], the plant P must be expressed in real factored form:

$$P(s) = \frac{K \cdot e^{-\tau s} \cdot \prod_{i=1}^{m}(s+z_i) \cdot \prod_{j=1}^{a}\left(s^2 + 2 \cdot \delta_j \cdot \omega_{0j} + \omega_{0j}^2\right)}{s^N \cdot \prod_{l=1}^{n}(s+p_l) \cdot \prod_{q=1}^{b}\left(s^2 + 2 \cdot \delta_q \cdot \omega_{0q} + \omega_{0q}^2\right)} \quad (1)$$

where $K,\ \tau,\ z_i,\ \delta_j,\ \omega_{0j},\ p_l,\ \delta_q,\ \omega_{0q}$ are independent variables which can take the following uncertainty in their value:

$$K \in \left[K_{\min}, K_{\max}\right] \subset \Re^- o \Re^+ \quad (2)$$

$$\tau \in \left[\tau_{\min}, \tau_{\max}\right] \subset \Re^+ \quad (3)$$

$$z_i \in \left[z_{i\min}, z_{i\max}\right] \subset \Re \quad i = 1,...,m \quad (4)$$

$$p_l \in \left[p_{l\min}, z_{l\max}\right] \subset \Re \quad l = 1,...,n \quad (5)$$

$$\delta_j \in \left[\delta_{j\min}, \delta_{j\max}\right] \subset \Re^- o \Re^+ \quad j = 1,....,a \quad (6)$$

$$\omega_{0j} \in \left[\omega_{0j\min}, \omega_{0j\max}\right] \subset \Re^+ \quad j = 1,...,a \quad (7)$$

$$\delta_q \in \left[\delta_{q\min}, \delta_{q\max}\right] \subset \Re^- o \Re^+ \quad q = 1,....,b \quad (8)$$

$$\omega_q \in \left[\omega_{q\min}, \omega_{q\max}\right] \subset \Re^+ \quad q = 1,...,b \quad (9)$$

QFTIT implements the QFT design by considering a maximum of five design stages.
   1) Templates computation.
   2) Specifications
   3) Loop-shaping
   4) Pre-filter design
   5) Validation

When the user is working in a certain stage of the design, it is possible without difficulty to advance on to the next step or return to any of the previous phases.

QFTIT allows two different modes of execution which can be used to complement each other:

1) *Interactive mode*. The mouse pointer is used to operate over the different items displayed in the window. This mode of execution clearly shows the interactivity amongst the different items displayed.

2) *Dialog mode*. QFTIT tool has a Settings menu that includes a total of thirty entries divided in nine groups. Once a certain option of the Settings menu has been selected, a dialog box must be filled-in. This mode of execution, which is faster than the first mode, is less interactive.

In the next sections we are going to present the main features of these stages from the QFTIT point of view.

### III. TEMPLATES COMPUTATION

A template $\mathcal{T}(\omega_i)$ is a set of complex numbers representing the frequency response of an uncertain plant at fixed frequency $\omega_i$. During this stage, the user defines the plant (1), by configuring the uncertainty of its components. Furthermore, the user also selects the set of trial frequencies $\Omega$.

$$\Omega = \left\{\omega_1, \omega_2,..., \omega_N\right\} \qquad N \leq 10 \qquad (10)$$

The templates $\mathcal{T}(\omega_i)$ $\omega_i \in \Omega$ are simultaneously computed in an interactive way and showed on a Nichols chart while user does these actions.

### IV. SPECIFICATIONS

In this stage, the users select and configure the specifications (see Table 1) that their design must fulfil. For each selected Type $i$ specification the user must configure the value of its associated $W_i(s)$, by adding zeroes and poles For this objective it is necessary to select the frequencies under which each specification must be verified. There is also a simultaneous generation on a Nichols chart of associated bounds for each specification while user does these actions. Bounds are magnitude constraints on a nominal open-loop function that are equivalent to closed-loop specifications. The intersection of the different bounds generates the worst case bounds $B(\omega_i)$ $\omega_i \in \Omega$ in each frequency. The user can choose how the bounds are shown: individual grouped or intersected.

TABLE I
SPECIFICATIONS IMPLEMENTED IN QFTIT

| Type | Specification |
|---|---|
| 1: Robust stability | $\left\|\dfrac{P \cdot C}{1 + P \cdot C}\right\| \leq W_{s1}$ |
| 2: Disturbances rejection at plant output | $\left\|\dfrac{1}{1 + P \cdot C}\right\| \leq W_{s2}$ |
| 3: Disturbances rejection at plant input | $\left\|\dfrac{P}{1 + P \cdot C}\right\| \leq W_{s3}$ |
| 4: Control effort | $\left\|\dfrac{C}{1 + P \cdot C}\right\| \leq W_{s4}$ |
| 5: Tracking bandwidth | $\left\|\dfrac{P \cdot C}{1 + P \cdot C}\right\| \leq W_{s5}$ |
| 6: 2-DOF tracking | $W_{s6a} \leq \left(\left\|\dfrac{P \cdot C}{1 + P \cdot C}\right\|\right) \leq W_{s6b}$ |

### V. LOOP-SHAPING

During this phase, the user performs the synthesis of the controller $C(s)$ by shaping the open-loop transfer function

$L_0(s)=P_0(s)\cdot C(s)$ in the Nichols diagram. The user can see the motion of the controller's zeroes and poles over the complex plane and the variation of its symbolic transfer function when $L_0$ is modified in the Nichols diagram. Besides it is possible to observe the change of the shape of the $L_0$ in the Nichols diagram and the variation of the expression of the controller's transfer function when any movement, addition or suppression of its zeroes or poles in the complex plane is done.

## VI. Pre-Filter Design

In this step the user performs the synthesis of the pre-filter $F(s)$ if the Type 6 *2-DOF tracking* specification has been previously activated by shaping of the minimum and maximum values of the closed-loop transfer function in the Bode magnitude diagram. The features of this phase are very similar to the previous stage.

## VII. Validation

In this stage the users verify that the specifications on the frequency domain of their proposed designs are fulfilled in Bode diagram. User can also see the changes that take place in the temporary representation of the manipulated variable and in the controlled variable due to the variation of the nominal values of the different elements of the plant. Besides the user can configure the magnitude and the occurrence instant of a step perturbation in the input of the plant and see its effects on the manipulated and controlled variables.

## VIII. An Illustrative Example

In this section an illustrative robust regulation problem is solved using QFTIT. The proposed example is the same used in [6] to describe FDCDT. Thus, a habitual user of FDCDT will be able to easily perceive the QFTIT advantages and limitations.

### A. Statement of the regulation problem

Let the family of plants

$$\mathcal{P} = \left\{ P(s) = \frac{K}{(s+a)\cdot(s+b)} \atop K \in [1,10],\ a \in [1,5],\ b \in [20,30] \right\} \quad (11)$$

The nominal values are: $K_0=1$, $a_0=5$ and $b_0=30$. A controller $C(s)$ must be obtained in order to the closed-loop system fulfill the following specifications:

--Type 1: *Robust stability*. The desired phase margin (PM) must be greater or equal to 49.25 $\forall \omega \geq 0$. An equation that relates PM with $W_{s1}$ is given in [11]:

$$W_{s1} = \sqrt{\frac{0.5}{\cos\left(\pi\cdot\left(1-\frac{PM}{180^\circ}\right)\right)+1}} \quad (12)$$

Evaluating this expression with PM=49.25 yields $W_{s1}$=1.2.

--Type 2: *Disturbances rejection at plant output*

$$W_{s2} = 0.02\frac{(s+6)\cdot(s+8)\cdot(s+50)}{s^2 + 2\cdot0.5\cdot13\cdot s + (13)^2} \quad (13)$$

$\forall\ \omega \in [0,10]$(rad/s).

--Type 3: *Disturbances rejection at plant input* $W_{s1}$=0.01 $\forall\ \omega \in [0,50]$(rad/s).

### B. Stage 1: Templates computation

According to the specifications of this problem a possible set of trial frequencies (rad/s) is

$$\Omega = \{0.1, 5, 10, 100\} \quad (14)$$

The frequencies $\Omega$ can be introduced in QFTIT using its area *Template frequency vector* (see Figure 2). There is a horizontal axis $\omega$ representing radian per seconds. It is possible to add, remove and change the frequencies of $\Omega$. Each of these frequencies is represented by a vertical segment with an associated colour code that can be moved along the $\omega$ axis.

The elements of the plant (11) are a gain and two real-poles. One possible way of introducing them into QFTIT is using their areas (see Figure 2) *Operations over plant P* and *Uncertainty plant description*.

The area *Operations over plant P* is used to select the type of plant element (real-pole, real-zero, complex-pole, complex-zero, integrator) on which we want to perform some type of action (move, add or remove) in the *Uncertainty plant description* area. It is also possible to configure the delay and the gain of the plant by using two sliders. For the plant (11) the slider associated with the gain would have to be moved in order to configure its minimum value $k_{min}$= 1, its maximum value $k_{max}$=10 and its nominal value $k_{nom}$=1.

The *Uncertainty plant description* area is used to graphically design the configuration of the uncertainty of the plant poles and zeroes. This operation is carried out with the use of the mouse over the selected pole or zero element. For simple zeroes or poles the uncertainty is represented by a segment, whilst for complex zeroes and poles it is represented by a circular sector limited by the maximum and minimum values of the damping factor and the natural frequency of each complex item (pole or zero). Both representations include the extreme values as well as the nominal value.

For the plant defined in (11), by selecting the adequate options in the *Operations over plant P* area, it would be possible to add two real-poles ($s=-a$, $s=-b$) in the *Uncertainty plant description* area and to configure the uncertainty ($a$, $b$) of these elements and their nominal values (12) by dragging the mouse.

The area *Templates* shows a Nichols diagram that includes the templates calculated for the set of frequencies defined in $\Omega$.
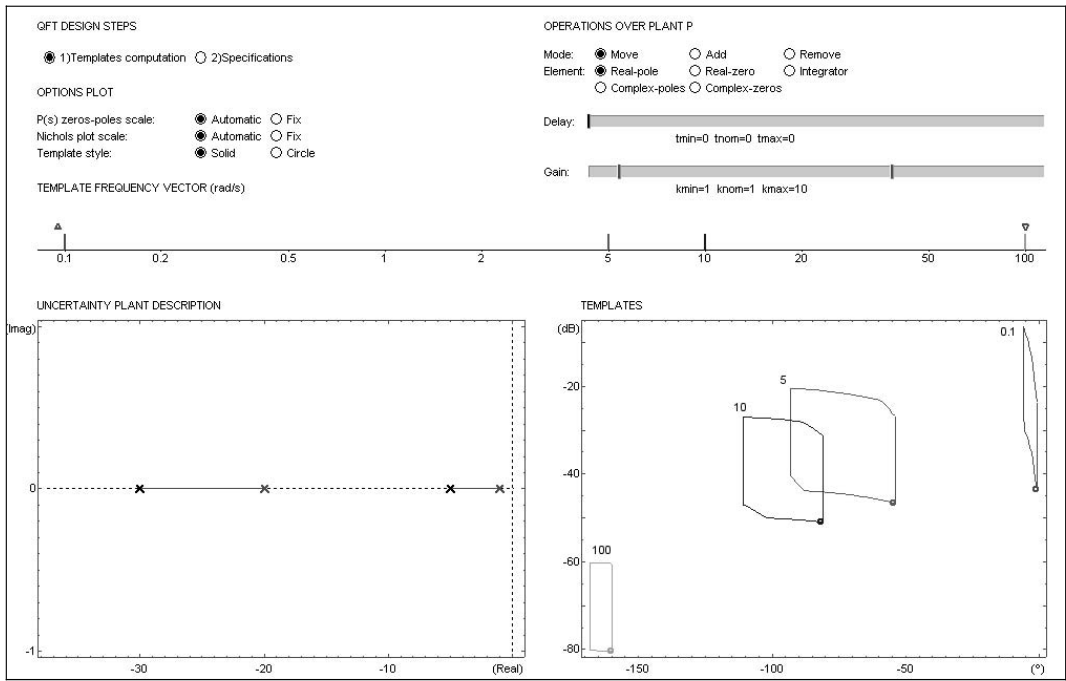
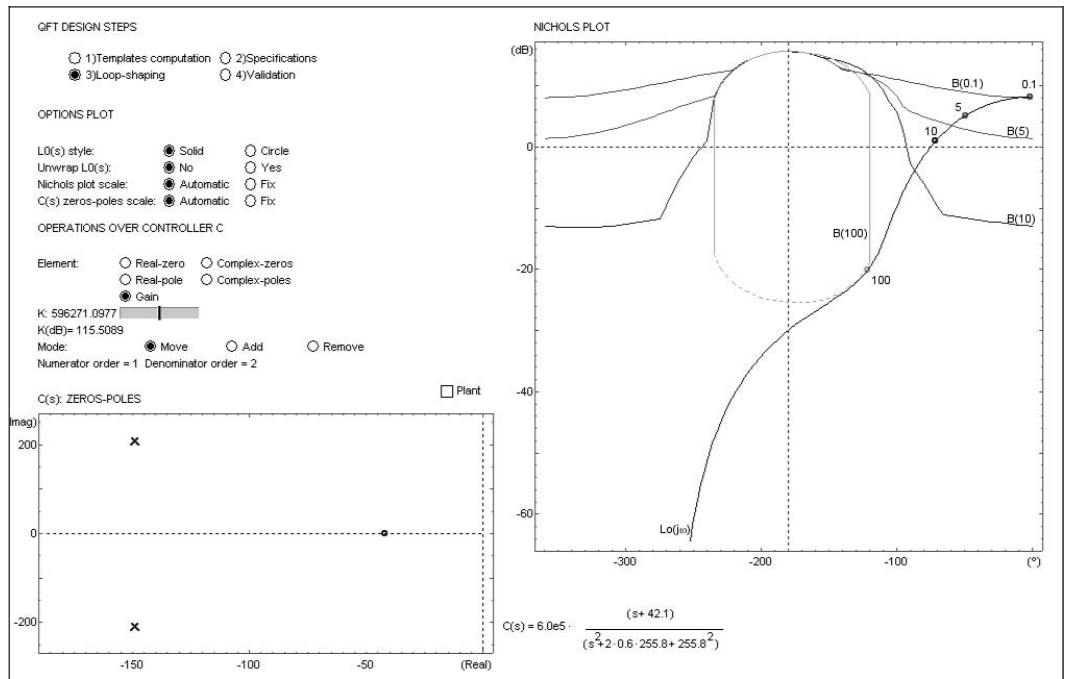Fig. 2. Aspect of the QFTIT window after stage 1 (Templates Computation) for the proposed problem.



Fig. 3. Aspect of the QFTIT window after stage 3 (Loop-Shaping) for the proposed problem.

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For this problem (see Figure 2) it shows four templates, each with the colour associated with the corresponding frequency in the set Ω.

### C. Stage 2: Specifications

The proposed problem has three specifications: robust stability specification (Type 1), disturbance rejection at plant output (Type 2) and disturbance rejection at plant input (Type 3). Type 1 specification can easily be configured. First, the user must select and activate this kind of specification in the *Specification type* area (see Figure 4). In the *W(s) frequency domain specification* area it is possible to configure then the value of the constant $W_{s1}$ by simply dragging a slider from value 1 to the desired value 1.2. Just under the slider there is a display showing the value of the gain margin (GM≥ 1.8) and the phase margin (PM≥ 49.2°) obtained. It is also possible to view simultaneously and interactively how the specification modulus is being modified in the Bode diagram and how the associated bounds change in the Nichols diagram.
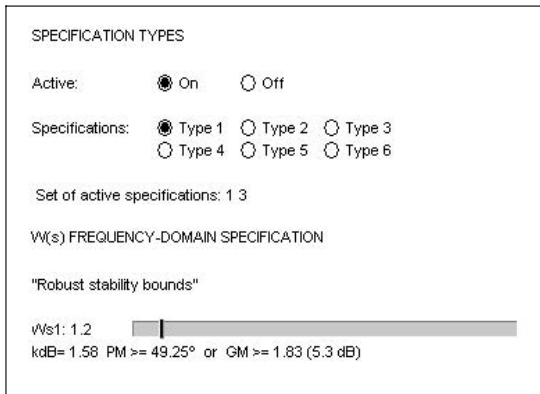


Fig. 4. Aspect of the *Specification Types* and *W(s) Frequency-Domain Specification* areas of the program QFTIT during the configuration of the robust stability specification for the proposed problem.

Type 2 specification is given in the form of the transfer function (13). QFTIT easily allows configuring a transfer function $W(s)$ associated to a certain specification. A possible interactive form is to select in the *W(s) frequency domain specification* area (see Figure 5) the type of element of the transfer function $W(s)$ (real-pole, real-zero, complex-pole, complex-zero, integrator) on which we want to execute a certain action (move, add or remove) within the *W(s): Zeros-Poles* zone. In this area, QFTIT displays a map of poles and zeroes that allows a configuration by means changing the value with the mouse of the transfer function $W(s)$ associated to a certain specification. It is possible to add, move and suppress the zeroes and the poles of $W(s)$.

For the transfer function (14) it would be necessary to configure a pair of complex poles with natural frequency $\omega_0$=13 (rad/s) and damping factor δ=0.5, and three real zeroes in -6, -8 and -50. The gain of the transfer function can be configured dragging from 1 to 0.02 the *Kw* slider placed in the *W(s) frequency domain specification* area.

Type 3 specification can be configures in a similar way to Type 1. In this case the slider must be dragged from value 1 to the desired value 0.01.
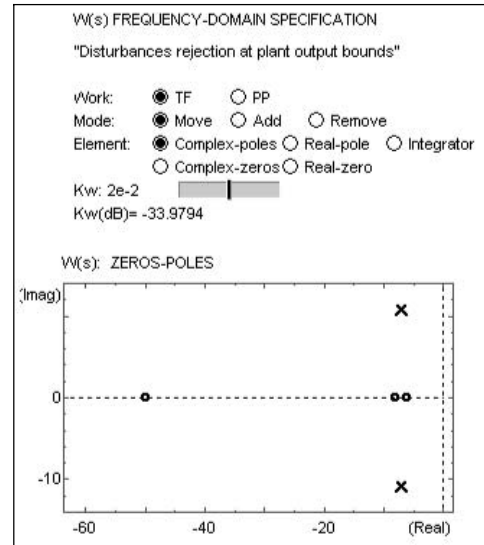


Fig. 5. Aspect of *W(s) Frequency-Domain Specification* and *W(s): Zeros-poles* areas of the program QFTIT during the configuration of the robust stability specification for the proposed problem.

### D. Stage 3: Loop-shaping

During this design stage, the main manipulation that the user can perform is the displacement of the open-loop transfer function $L_0$ in the *Nichols Plot* area (see Figure 3) in certain directions depending on the selected controller item in the *Operation Over Controller C zone*. Simultaneously, the zeroes and the poles are displaced in the *C(s) Zeros-Poles* zone. Besides the user can displace the zeroes and the poles of the controller in the *C(s) Zeros-Poles* zone and see the displacement of $L_0$ in the Nichols plot. Thus, the user has a very interactive and flexible tool to perform the synthesis of the controller.

Figure 3 displays the aspect of the QFTIT window after the Loop-Shaping stage. The Nichols diagram shows the intersection of the bounds $\mathcal{B}(j\omega_i)$ $\omega_i \in \Omega$ associated with the established specifications and the final $L_0$. It can be observed how the points $L_0(j\omega_i)$ are placed above the boundaries $\mathcal{B}(j\omega_i)$.

The expression of the designed controller is

$$C(s) = 5.96 \cdot 10^6 \frac{s + 42.1}{s^2 + 2 \cdot 0.6 \cdot 255.8 \cdot s + (255.8)^2} \quad (15)$$

It is a controller with one real-zero and a pair of complex-poles. The zeroes and the poles of $C(s)$ are represented in the *C(s) Zeros-Poles* zone.

### E. Stage 4: Validation

During this design stage, the user only has to select the type of specification to validate in the *Specification Types zone*, and QFTIT immediately shows the modulus of $W_{si}$ and the worst case modulus of the associated characteristic function of the system (see Table I) in a Bode Magnitude diagram placed in the *Validation* area.

For the proposed problem three specifications have to be validated. Figure 6 shows the maximum magnitude in dB of the closed-loop transfer function $max\{|L(j\omega)/(1+L(j\omega))|\}$ and the constant gain line $W_{s1}$=1.2 (1.58 dB). As $max\{|L(j\omega)/(1+L(j\omega))|\}$ does not surpass the

horizontal line in any of the frequencies, the robust stability specification is fulfilled.

On the other hand, Figure 7 displays $W_{2s}(j\omega)$ (broken line) and the maximum magnitude of the sensitivity function $max\{|1/(1+L(j\omega))|\}$ (solid line). It can be observed how the specification of disturbance rejection at plant output is fulfilled, since $max\{|1/(1+L(j\omega))|\}$ is below $W_{s2}(j\omega)$ in any of the frequencies.

Finally, Figure 8 shows the maximum magnitude in dB of the transfer function $max\{|L(j\omega)/(1+L(j\omega))|\}$ and the constant gain line $W_{s3}$=0.01 (1.58 dB).As $max\{|L(j\omega)/(1+L(j\omega))|\}$ does not surpass the horizontal line in any of the frequencies, the specification is fulfilled.
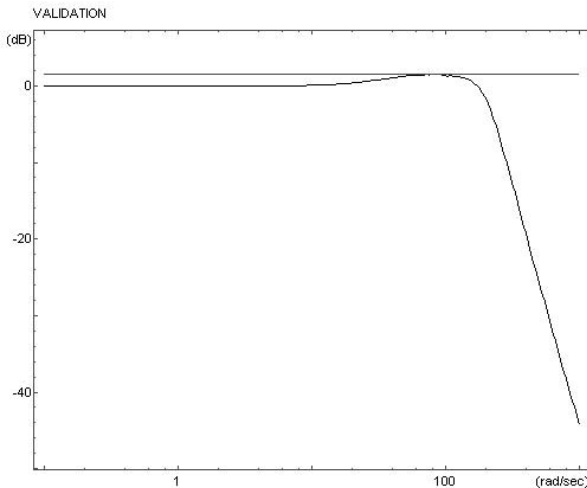


Fig. 6. Maximum magnitude in dB of the closed-loop transfer function $L(j\omega)/(1+L(j\omega))$ and the specification $W_{s1}$(horizontal line).
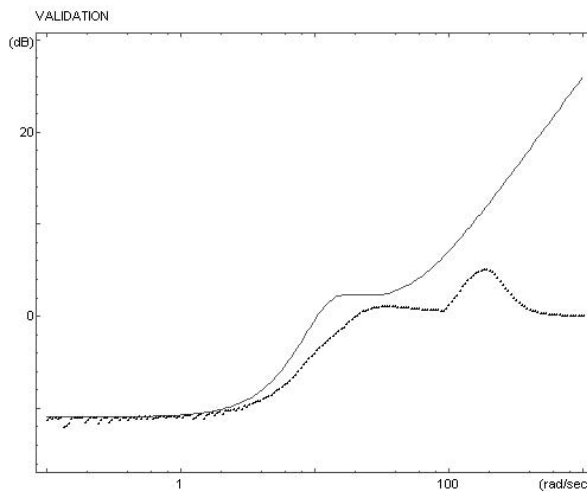


Fig. 7. Maximum magnitude in dB of the sensitivity transfer function $1/(1+L(j\omega))$ (broken line) and the specification $W_{s2}$ (solid line).
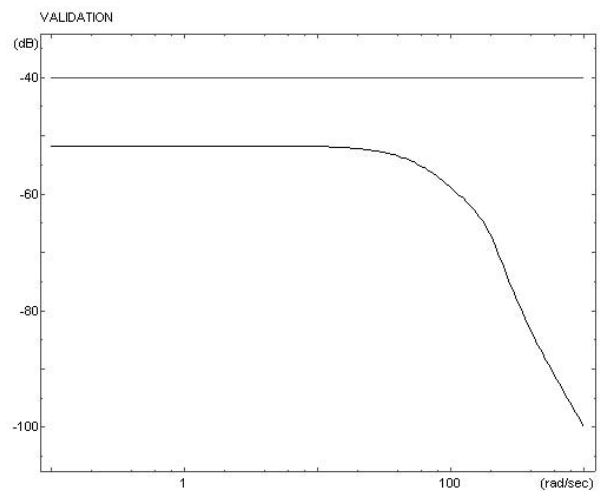


Fig. 8. Maximum magnitude in dB of transfer function $P/(1+L)$ and the specification $W_{s3}$ (horizontal line).

## IX. CONCLUSION

In this work a new interactive software tool to design robust controllers using the QFT methodology has been presented. An illustrative example has been solved using this tool in order to explain its ease of use. Obviously, the present version of QFTIT has some limitations. In future tool versions will be included other types of plant definition, other templates calculation algorithms, other types of specifications, etc. The authors are pleased about receiving any suggestions in order to improve the tool.

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