AN ITERATIVE METHOD FOR TUNING DECENTRALIZED PID CONTROLLERS

F. Vázquez, F. Morilla, S. Dormido

fmorilla@dia.uned.es. Dpto de Informática y Automática.Facultad de Ciencias.UNED.
C/ Senda de Rey s/n 28040 Madrid (SPAIN)

Abstract: In this paper a method of tuning decentralized PID controllers for multivariable systems is presented. It is based on an iterative numeric algorithm, and it uses information coming from the frequency response of the full matrix of the system transfer functions. Then, the effects of the interaction are included in the design because the off-diagonal elements in this matrix are taken into account. The aim is to meet the design specifications that were gain margins, phase margins or a combination of both in the different experiments carried out. Three examples are shown: two obtained from the literature and one from a real plant. The results are compared with another type of MIMO tuning.

Keywords: PID controllers, multivariable control systems, gain and phase margins.

1. INTRODUCTION

Although numerous techniques of advanced control exist (robust, predictive, adaptative control, etc) the decentralized control is still frequently used in industry as a strategy of multivariable control, as a mere control technique or used in the most internal control loop and with some of the previous ones in a superior level. The sacrifice that supposes the invariable deterioration of the benefits of a decentralized control structure when is compared with a full multivariable control strategy, is compensated with certain advantages like design and hardware simplicity or easiness of use.

However, the use of this type of strategies is not always possible due to the interaction effects. A severe interaction will produce a loss on closed-loop performance and stability. In these cases it will be necessary to give up the use of these techniques and of think in another type of strategies commented previously. In order to study these interaction effects numerous methods exist, one of most often used in industrial applications being the relative gain matrix (RGA) (Bristol,1966) with their numerous variants, due to their calculation simplicity. The use of this matrix is not only limited to the steady state analysis to solve the pairing problem among variables. Furthermore, it is supplemented with other calculations to provide indexes of stability, or interaction dynamic analysis. In this line, methods like the direct and inverse Nyquist's arrays (DNA and INA), (Rosenbrock,1979; Maciejowsky,1989), supplemented with Gershgorin's bands, and other calculations like the condition numbers, Niederlinsky's index, dynamic RGA, etc, are frequently used.

Once this type of decentralized techniques has been selected for use, either because the interaction is not severe or because only some of the system outputs are needed, and once the pairing problem has been solved, the following step will be choosing the controller and its tuning. Most of the papers published in this field use PID controllers (with their different possibilities) for a multitude of well-known reasons: industrial implantation, robustness, employment easiness... Some methods found in the literature could be classified under the following headings that show the interest that the topic has raised in the last years:

Tuning methods based on heuristic formulas: These formulas indicate the direction in which the parameters have to be detuned to compensate the interaction effects when all the loops are closed. The initial parameters have been previously calculated using some SISO technique, that is, making a design with only the elements of the main diagonal (the g_{ii}) of the system transfer function matrix. (Shinskey,1995; McAvoy, 1983) works are in this line.

Designs based on the relay method: These techniques are able to obtain the different parameters of the controllers from the frequency and amplitude of a maintained oscillation, achieved with the relay when closing the loops progressively. Numerous iteration algorithms exist, like those of (Wang et al, 1996 and 1997; Menani and Koivo, 1996; Halevy et al, 1996; Shiu and Hwang, 1998) to mention some.

True multivariable tuning methods: In this case, in order to calculate the controller parameters, these methods do not use only a part of the information of the transfer function matrix but rather in the design they already take into account the interaction of the off-diagonal elements. One of the most outstanding is the work of Ho et al (1996), that provides on-line tuning formulas to obtain the PID controllers for two by two systems. This is a method that uses Gershgorin's band to provide a correction to the specifications of phase and gain margins and then they enter in a SISO formulation.

The present work could be framed within this last group. In it, an iteration algorithm is proposed to achieve the PID parameters for any multivariable system, paying special attention to those of two inputs and two outputs. First, the tuning method is exposed presenting the notation and the used algorithm. Later the method is applied to three examples: two taken from literature and one developed by the authors.

2. ITERATIVE METHOD FOR TUNING DECENTRALIZED CONTROLLERS

Let suppose a MIMO system with two inputs and two outputs. The use of a decentralized control strategy has been decided and the pairing problem of variables has been solved. It is aimed to control this system by means of two controllers k_1 and k_2. The controller k_i closes a loop between the controlled variable y_i and the manipulated variable u_i. If the controller k_2 has already been tuned by some method and it is sought to make the same thing with the k_1, as can be seen in Figure 1, between the controlled variable y_1 and the manipulated variable u_1 there is something else than the transfer function g_{11}, because an additive action appears. This is due to the existence of one of the hidden loops described in (Shinskey,1995). The effect of this additive action could be represented as the block a_1 shown in Figure 2. In the same figure, the combination of g_{11} and a_1 is denominated $\tilde{g}_{11}$, following the notation of Zhu and Chiu (1998) (in this paper a study of the interaction and stability characterization of this type of systems can be found).

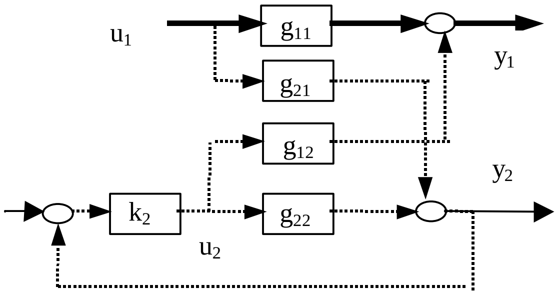

Fig. 1: Interaction of the hidden loop on the loop 1.

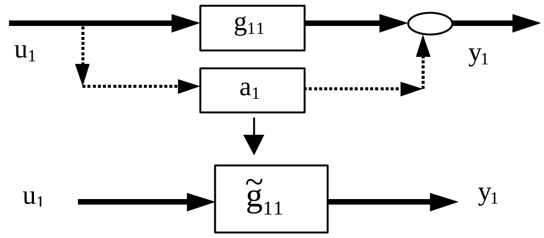

Fig. 2: Effect of the interaction and the final result

With this scheme in mind, the proposed tuning approach instead of calculating a controller for g_{11} (that would be when there is not interaction) will proceed to tune k_1 for $\tilde{g}_{11}$ which already incorporates the effect of this interaction. Nevertheless, the modification of k_1 will have its effect on the other loop, a new tuning of k_2 becoming necessary. This operation could be made successively until the producing changes in k_1 and k_2 are small and each loop meets the design specifications. The calculation could be carried out using some SISO tuning method adapted, as it will be seen later on.

3. TUNING APPROACH FOR PID CONTROLLERS

Let suppose that k_1 and k_2 are two PID controllers described by

$$k_1^i = Kp_1^i\left(1+\frac{1}{Ti_1^i s}+Td_1^i s\right)$$
$$k_2^i = Kp_2^i\left(1+\frac{1}{Ti_2^i s}+Td_2^i s\right) \qquad (1)$$

where the superscript indicates the number of the iteration, and the subscript the loop to which the calculation refers. The algorithm starts with k_1^o and k_2^o obtained exclusively for g_{11} and g_{22} by means of a SISO method or an arbitrary controller that could be Kp=1, Ti=9999, Td=0. The effect of this election has little repercussion in the number of iterations and in the convergence. The following iterative algorithm is proposed: from k_1^o, $\tilde{g}_{11}^o$ is obtained, and from k_2^o, $\tilde{g}_{22}^o$ is obtained, following the expressions:

$$\tilde{g}_{11}^i = g_{11} + a_1^i \qquad \tilde{g}_{22}^i = g_{22} + a_2^i \qquad (2)$$

$$a_1^i = -\frac{k_2^i g_{12} g_{21}}{1 + k_2^i g_{22}} \qquad a_2^i = -\frac{k_1^i g_{12} g_{21}}{1 + k_1^i g_{11}} \qquad (3)$$

If the couples $k_1^i\ \tilde{g}_{11}^i$ and $k_2^i\ \tilde{g}_{22}^i$ meet the design specifications the iteration concludes. Otherwise the iteration number increases.

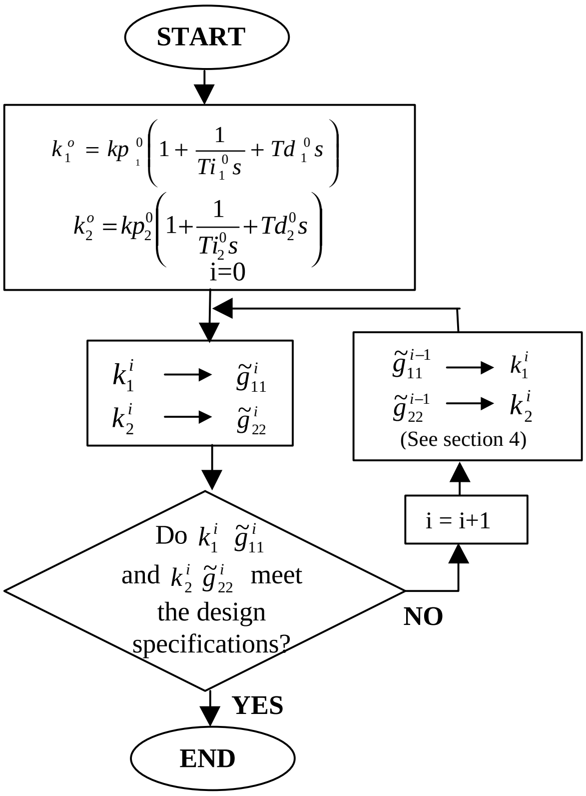

Fig. 3: Flow diagram of the algorithm

The controller k_1^i is then calculated for the model $\tilde{g}_{11}^{i-1}$ and the controller k_2^i for the model $\tilde{g}_{22}^{i-1}$. With the new controllers k_1^i and k_2^i, $\tilde{g}_{11}^i$ and $\tilde{g}_{22}^i$ are recalculated. With these data a new iteration begins. This process is shown in Figure 3.

4. OBTAINING THE k_j^i FROM THE $\tilde{g}_{jj}^i$

In order to obtain the k_j^i from the $\tilde{g}_{jj}^i$ a lot of SISO methods can be employed. However the calculation of the $\tilde{g}_{jj}^i$ from a_j^i is not immediate, due to certain problems: the presence of time delays in the functions g_{ij} can make the analytic calculation complicated. Not less important, the variety of types of models that can take place when making the quotient given in the expression of a_j^i makes it impossible to use a particular formula among the existing ones for models of first order plus delay, two time constant plus delay,..., (Ho et al, 1996). To solve these problems, the proposed method uses an adaptation of a generic tuning method allowing a numeric calculation instead of an analytic one, that is to say, not expressing the g_{ij} as a transfer function but as the array of frequency response of the transfer function. This operation mode is much easier and quicker and the $\tilde{g}_{jj}^i$ will be other response arrays, without an analytic expression in the form of a concrete transfer function.

An adaptation of the analytic method of Morilla and Dormido (1998) has been chosen. This is an extension of the Aström and Hägglund (1984) method consisting in a generalization of the Ziegler-Nichols formulas to move a point of Nyquist's diagram of the open loop transfer function from a position A (in controller's absence) until another position B (in controller's presence), as Figure 4 shows. If the controller is an ideal PID controller given by the following expression

$$k(s) = Kp\left(1 + \frac{1}{Ti\ s} + Td\ s\right) \qquad (4)$$

the determination of its parameters can be summarized in the equations

$$Kp = \frac{r_b \cos(f_b - f_a)}{r_a} \qquad (5)$$

$$Ti = \frac{1}{2\,a\,w_c}\left(tg\,(f_b - f_a) + \sqrt{4\,a + tg^2\,(f_b - f_a)}\right) \qquad (6)$$

$$Td = a\ \ Ti \qquad (7)$$

where ω_c is the frequency corresponding to the point A elected in the Nyquist's diagram, r_a and ϕ_a are the gain and the angle of this same point, r_b and ϕ_b the destination point B gain and angle, and α the ratio between the derivative and integral time constants, that should be specified.

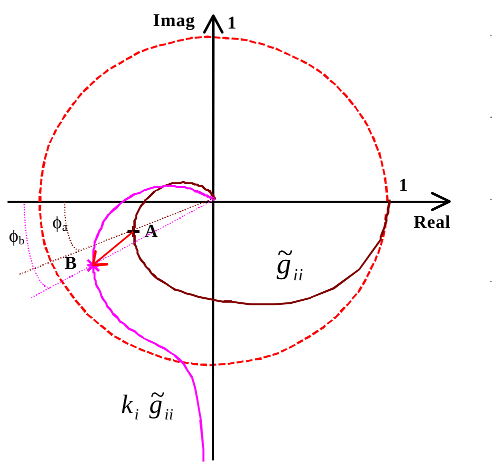

Fig. 4: Process and process plus controller Nyquist's diagram and circle unit. The controller is PID and it has been tuned to move the point A to the point B.

As Morilla and Dormido (1998) show, its design admit three design possibilities: phase margin (PM), gain margin (GM) and a combination of both. Due to space limitations, only the design for phase margin will be described. First of all, the frequency range ω_c that admits solution for a certain controller is calculated, that is to say, the set of points A, keeping in mind that the process included in the algorithm is a frequency response array, $\tilde{g}_{ii}$. From this array, the point A gain and angle, r_a and ϕ_a, are obtained. From the PM specifications, ϕ_b is calculated. Because point B is located on the unit circle, then $r_b = 1$. With these data, Kp and Ti are calculated from expressions (5) to (6) for each point A with possible solution. In order to choose one among all of the solutions, a selection approach is needed. For example, if it is sought to minimize the integral of the absolute value of the error (IAE), the maximum ratio Kp/Ti will be looked for. Parameter α can also be chosen to minimize some other optimization approach.

It is also necessary to notice that it is possible to use any other SISO tuning method whenever it is adapted to solve the problems commented before.

5. EXAMPLES

Example 1: As a first example, a water-methanol distillation column proposed by Ho et al (1996) is analyzed. The system is described by the following transfer function matrix

$$\begin{pmatrix} \frac{12.8}{16.7s+1}e^{-s} & \frac{-18.9}{21s+1}e^{-3s} \\ \frac{6.6}{10.9s+1}e^{-7s} & \frac{-19.4}{14.4s+1}e^{-3s} \end{pmatrix} \quad (8)$$

If a PM=20º and a GM=2 for both loops on Ho's tuning formulas are specified, the following parameters are obtained: Kp_1=0.57, Ti_1=20.7, Td_1=0, Kp_2=-0.11, Ti_2=12.88 Td_2=0; while with specifications of PM=30º and GM=3, for both loops, they are Kp_1=0.38, Ti_1=21.64, Td_1=0, Kp_2=-0.07, Ti_2=14.8 Td_2=0. However, if an analysis of these tunings is made the obtained results show that the values are very far from the initial specifications:

Tuning	Loop 1	Loop 2
1th	PM=51.6º GM=3	PM=94º GM=2.2
2th	PM=65º GM=5.3	PM=103º GM=3.8

This is because any design that uses Gershgorin's bands will provide very conservative tunings when the interaction is appreciable. The main reason is that in most of the cases, these bands are excessively wide and they produce a correction in the initial phase and gain margins. Gershgorin's bands for this system with the first tuning can be seen in the graph of Figure 5, superimposed with direct Nyquist array (DNA). In this graph $k_1\tilde{g}_{11}$ and $k_2\tilde{g}_{22}$ (on continuous line) are also superimposed, both being quite far from the band borders, which shows that any design that uses these bands in its calculation process will lead inevitably to extra-conservative tunings. A more exhaustive description of Gershgorin's bands and direct Nyquist's arrays can be found in Rosenbrock (1979) or Maciejowsky (1989).

Using the algorithm proposed in this work with specifications of PM_1=45 and PM_2=45 and beginning with Kp_1=1, Ti_1=9999, Td_1=0 y Kp_2=-1, Ti_2=9999, Td_2=0 the results of Table 1 are obtained for each iteration. It is observed that five iterations have been enough to meet the specifications in the two control loops.

Table1: Results obtained in each iteration for example 1 with specifications of PM_1=PM_2=45º

It.	PM_1	PM_2	GM_1	GM_2	Kp_1	Ti_1	Kp_2	Ti_2
1	0.08	0.09	7.55	1	0.22	2.57	-0.11	4.14
2	37	48	2.39	2.16	0.75	3.33	-0.04	1.67
3	42	40	2.63	1.41	0.70	4.25	-0.09	3.17
4	44	48	2.47	1.51	0.73	3.51	-0.09	3.10
5	45	45	2.48	1.46	0.73	3.56	-0.09	3.11

In Table 2 other tunings are showed, carried out for the PI controllers in example 1 with different sets of specifications: only PM, only GM or combinations of both.

Table 2: Tuning with different specifications for example

Specifications				Performance				Control parameters	
PM_1	PM_2	GM_1	GM_2	PM_1	PM_2	GM_1	GM_2	Kp_1, Ti_1, Td_1	Kp_2, Ti_2, Td_2
40	40	-	-	40	40	0.12	1.36	0.69, 2.88, 0	-0.08, 2.78, 0
40	60	-	-	40	60	2.43	1.43	0.73, 2.88, 0	-0.10, 4.41, 0
60	90	-	-	60	82	2.54	3.32	0.78, 15, 0	-0.06, 7.37, 0
-	-	4	4	20	55	4	4	0.32, 1.64, 0	-0.006, 0.6, 0
-	-	2	5	16	56	2	5	0.66, 1.66, 0	-0.015, 6.15, 0
45	80	4	3	46	80	4	3	0.47, 6.58, 0	-0.06, 6.15, 0
30	65	3	4	30	65	3	4	0.56, 2.49, 0	-0.029, 3.04, 0

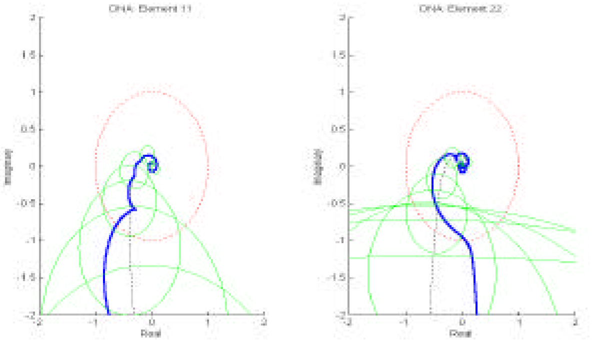

Fig.5: DNA with Gershgorin's bands superimposed for example 1 with the first tuning of Ho's work. $k_1\tilde{g}_{11}$ and $k_2\tilde{g}_{22}$ on continuous line and $k_1 g_{11}$ and $k_2 g_{22}$ in discontinuous

Example 2: It is a four-coupled tanks system, also proposed by Ho et al (1996) whose transfer functions matrix is given by

$$\begin{pmatrix} \dfrac{4.3}{383s+1}e^{-40s} & \dfrac{1.8}{383s+1}e^{-140s} \\ \dfrac{1.2}{281s+1}e^{-80s} & \dfrac{2.5}{281s+1}e^{-40s} \end{pmatrix} \quad (9)$$

By means of Ho's tuning formulas, if a PM_1=40º and a GM_1=4 for the first loop and a PM_2=30º and a GM_2=3 for the second are specified, the controllers have the following parameters: Kp_1=0.62, Ti_1=217, Td_1=0, Kp_2=0.94, Ti_2=193 Td_2=0. If these tunings are analyzed, the obtained results (PM_1=67º, GM_1=5.35 for the first loop and PM_2=61.5º, GM_2=4.5 for the second) are both far from the specifications, although not as much as in the previous example because Gershgorin's bands are not as wide as in the previous example.

Using the iterative algorithm, designs shown in Table 3 can be obtained, where a great proximity between the specifications and the results can be observed, either in tunings for PM, by GM or combined of PM and GM, and using PI or PID controllers.

Ex*ample 3:* Lastly, in Table 4 some tunings for the control of a heat exchanger (Morilla and Vázquez, 1997) are shown. The process transfer function matrix is given by the following expression, and its scheme is shown in Figure 6.

$$\begin{pmatrix} T_1 \\ T_2 \end{pmatrix} = \begin{pmatrix} \dfrac{0.28}{638s+1}e^{-200s} & 0.021\cdot\dfrac{6000s+1}{1400s+1} \\ \dfrac{0.49}{824s+1}e^{-200s} & \dfrac{-0.19}{250s+1}e^{-80s} \end{pmatrix} \begin{pmatrix} Q \\ N_2 \end{pmatrix} \quad (10)$$

Due to the width of Gershgorin's bands (as it can be seen in Figure 7), it was impossible to tune the controllers using Ho's tuning formulas. However, Table 4 shows that the iterative algorithm allows multiple possibilities, most of them tested later in real experiences. Here the difference between the different tuning could be seen.

Table 3: Tuning for different specifications of example 2.

Specifications				Performance				Control parameters	
PM_1	*PM_2*	*GM_1*	*GM_2*	PM_1	PM_2	GM_1	GM_2	Kp_1, Ti_1, Td_1	Kp_2, Ti_2, Td_2
40	30	-	-	40	30	2	1.28	0.88, 101, 0	3.25, 240, 0
67	61	-	-	67	61	5.6	4.9	0.59, 209, 0	0.85, 173, 0
45	45	3	3	45	45	3	3	1.07, 208, 0	1.39, 192, 0
40	30	4	3	40	30	3.8	4.2	0.19, 48, 0	1.24, 91, 0
-	-	6	6	10	10	6	6	0.56, 55, 9.44	0.71, 54, 9.14

Table 4: Tunings for different specifications of example 3.

Specifications				Performance				Control parameters	
PM_1	*PM_2*	*GM_1*	*GM_2*	PM_1	PM_2	GM_1	GM_2	Kp_1, Ti_1, Td_1	Kp_2, Ti_2, Td_2
45	45	-	-	45	45	1.97	4.64	4.8, 436, 0	-6.13, 426, 0
30	70	-	-	30	70	1.85	9.64	6.38, 426, 0	-2.19, 280, 0
70	30	-	-	70	30	2.02	2.55	3.89, 627, 0	-10.1, 243, 0
-	-	3	3	35	27	3	3	0.76, 91, 0	-5.85, 97, 0
-	-	2	5	32.6	27.3	2	5	1.78, 175, 0	-3.71, 111, 0
45	45	-	-	43	47	2.15	3.45	7.1, 554, 77	-2.3, 132, 75
45	45	3	3	45	46	2.89	2.99	2.63, 263, 0	-9.91, 321, 0
45	45	3	3	45	43	3.12	3	2.73, 230, 12	-13.1, 231, 24
60	35	-	-	60	36	1.75	2.36	6.31, 528, 53	-9.47, 271, 27
75	60	5	6	74.7	61	4.8	6.06	2.1, 585, 0	-4.5, 250, 0

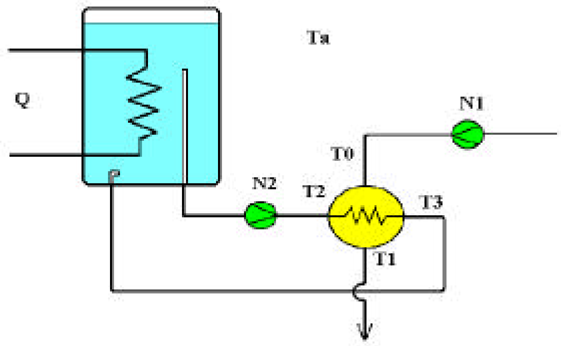

Fig. 6: Process scheme of example 3.

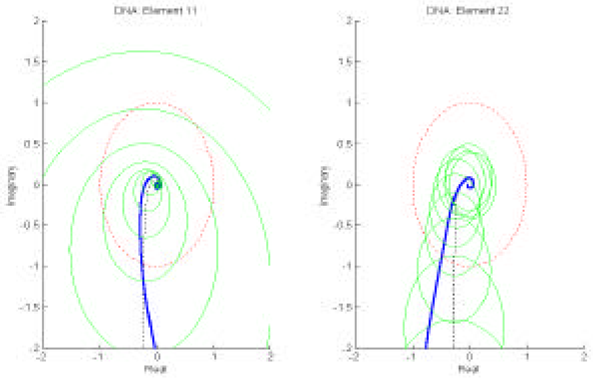

Fig. 7: DNA with Gershgorin's bands superimposed for example 3

In Figure 8 a graph of the real temporal response of the two temperatures is shown, where the set points of T_1 (output process temperature) were changed. The control parameters correspond to a conservative design, with the characteristics of the last row of Table 4.

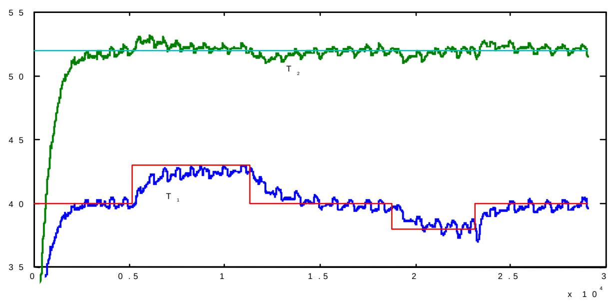

Fig. 8: Time response of the system of example 3 under control

CONCLUSIONS

In this paper, a new multiloop controller tuning method has been presented. The method, based on an iteration algorithm, has been compared with some of the examples used in the literature of MIMO tuning, achieving designs nearer to the specifications. Inside the algorithm any generic method of SISO controllers tuning could be used but with some variation. The difficulty that present some design methods that use Gershgorin's bands has also been shown, because with these, very conservative designs appear in the best of the cases, since in other they don't obtain a solution. In contrast, the method has been successfully applied to tuning the PID controllers of several real systems, like a heat exchanger.

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