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LOWER AT AUCTION: LOGICAL DETERMINATION

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LOWER AT AUCTION: LOGICAL DETERMINATION

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SUMMARY / ABSTRACT

The problem of the determination of the low amount in a public auction has to deserve a special attention for any bidder, especially in the case of the companies which come to the different public contests for the adjudication of several works and services, specifically those that refers to the undertaken constructors. From the study done, with a resolution of a practical example, it is deduced that the application of the diverse own criteria of the Theory of the Decision and the Theory of Games can drive to the adoption of the different strategies by the undertaken bidder. However, if the probability to award the work is the same, why has to be offered a determinate discount if it is possible offer a lower discount since the probabilistic point of view? Besides, if we do the mentioned before, it can be reached a bigger profit for the company contestant.

Key words: matrix, row, column, decision, game, mathematical hope, risk, uncertainty, probability.

RESUMEN

El problema de la determinación de la cuantía de las bajas en una subasta pública debe merecer especial atención para cualquier licitador, especialmente en el caso de las empresas que concurran a los diferentes concursos públicos para la adjudicación de diversas obras y servicios. Muy especialmente por lo que se refiere a las empresas constructoras. Del estudio desarrollado, con la resolución de un ejemplo práctico, se deduce que la aplicación de los diversos criterios propios de la Teoría de la Decisión y la Teoría de Juegos pueden conducir a la adopción de estrategias diferentes por parte de la empresa licitadora. Ahora bien, si la probabilidad de adjudicarse el concurso es la misma, ¿por qué se tiene que ofertar una rebaja determinada si es posible ofertar una rebaja inferior desde el punto de vista probabilístico? Además, con ello, se puede llegar a obtener un mayor beneficio para la empresa concursante.

***Palabras clave**: matriz, fila, columna, decisión, juego, esperanza matemática, riesgo, incertidumbre, probabilidad.*

RESUM

El problema de la determinació de la quantia de les baixes en una subhasta pública ha de merèixer especial atenció per a qualsevol licitador, especialment en el cas de les empreses que concorrin als diferents concursos públics per a l'adjudicació de diverses obres i serveis. Molt especialment pel que es refereix a les empreses constructores. De l'estudi desenvolupat, amb la resolució d'un exemple pràctic, es dedueix que l'aplicació dels diversos criteris propis de la Teoria de la Decisió i la Teoria de Jocs poden conduir a l'adopció d'estratègies diferents per part de l'empresa licitadora. Ara bé, si la probabilitat d'adjudicar-se l'obra és la mateixa, perquè ha d'oferir-se una rebaixa determinada si és possible oferir una rebaixa inferior des del punt de vista probabilístic? A més, amb això, es pot arribar a obtenir un major benefici per a l'empresa concursant.

***Paraules clau**: matriu, fila, columna, decisió, joc, esperança matemàtica, risc, incertesa, probabilitat.*

RÉSUMÉ

Le problème de la détermination du nombre d'adjudication à la baisse dans une vente aux enchères publique mériterait une attention particulière de la part de tout soumissionnaire, en particulier dans le cas des entreprises qui participent aux divers appels d'offres publics pour l'attribution de divers travaux et services. Très spécialement en ce qui concerne les entreprises de construction. De l'étude développée, avec la résolution d'un exemple pratique, on peut déduire que l'application des différents critères de la Théorie de la Décision et de la Théorie des Jeux peut conduire à l'adoption de différentes stratégies par la société soumissionnaire. Maintenant, si la probabilité de gagner le travail est la même, pourquoi devez-vous offrir une certaine réduction s'il est possible d'offrir une réduction plus faible du point de vue probabiliste? En outre, avec cela, vous pouvez obtenir un plus grand bénéfice pour l'entreprise concourante.

***Mots-clés:** matrice, rangée, colonne, décision, jeu, espoir mathématique, risque, incertitude, probabilité.*

1. INTRODUCTION

In the business world, as also happens in life itself, any business decision presupposes having to choose between alternative actions; if there were no different alternatives, there would be no decisions to be made: it would only be necessary to proceed with the only possible action. Consequently, a certain freedom of choice is involved in every business decision, which makes it necessary for the decision-maker to know what the possibilities are. Now, would you have to enter them into the model of the situation you have built, or do you have to build a separate model for each alternative? The answer to this question will depend on the problem in question; however, the need to ask yourself these questions, before making any decision, is a matter of principle that the company director can never forget.

All this leads us to the following consideration: since all the problems that are presented to the decision-maker involve at least two different alternatives, it is assumed that he has to choose the best one that is offered to him. Otherwise, how to recognize which is the best and how to do it quickly? It is important that the decision maker does not accept the first solution that lies ahead, be it good or bad. It is also important to realize that the time that the business manager has to act, and the company itself, will have been unfortunately lost if there is a long search among the possible alternatives to find the best one. After all, only a single alternative can be chosen, and some may even be very unpleasant. We are, therefore, facing a problem that could be called "search". What we need, then, is a rule that allows the decision maker to know the optimal degree of search corresponding to each situation, that is, what is the minimum number of alternatives to consider (Franquet, 2012).

Currently, it is not possible to formulate a simple rule of this kind to solve the problem. But what can be done is to draw the attention of the business manager to this issue. As is sometimes the case with our knowledge of business problems, the most to wait for is understanding the problem; the person in charge or decision-maker needs to realize that there is a search problem and that it is of the cost-benefit type. This means, obviously, that it will only be necessary to invest resources in the search for alternative actions to the extent that the benefit to be obtained exceeds the cost of the aforementioned investigation.

The problem of determining the amount of casualties in a public auction is a clear example of what has just been exposed and should deserve special attention for any bidder, especially in the case of companies that participate in different public tenders to the award of various works and services. Very especially with regard to construction companies.

There are, fundamentally, two criteria or reasons that justify this importance, namely:

a) A significant drop negatively influences the economic policy of society, if the income from certification of work or service is lower, equal or insufficiently higher ("opportunity costs") than the net costs of work.

b) On the other hand, it is often convenient - as far as possible and feasible- to lower the bidding price as much as possible, given the large number of competing contractors who can attend the auction, some of them with high and proven returns of his work, and others with the pure and simple desire to award the work or service as it may (at any price).

If the number of bidding companies is high enough (at least four or five), company A can decide - with notable and rational guarantees of success - the percentage of discount that it should offer in its escrow, using what has been given in calling "game against nature" ("General Theory of Strategy Games", which is a technique of applied Operational Research). In this case, "nature" is precisely the set consisting of, v. gr., the remaining four companies considered.

It should be borne in mind that here we will try to find out the optimal withdrawal in the auction that must be carried out by A in *ceteris paribus*[1] circumstances, that is, all bidders having satisfactorily complied with the other legal-administrative conditions required in the public tender in question and having obtained, therefore, an equal or similar score, in such a way that the proposal of decision of the award, on the part of the contracting table, will depend, solely, on the offered percentage of reduction in the auction on the part of the concurrent companies.

2. METHODOLOGY

Game and Decision Theories are one of the basic techniques of Operations Research whose operational management models may be radically different from those used in other techniques of that scientific discipline. It is about making one or more decisions in front of one or more adversaries whose reactions, when decisions are made, are little or nothing known. They constitute problems in which decisions are made in a situation of concurrence, with partial control of the final result and where the actors have opposite interests (since the benefits of some usually represent losses for the others). They try to guess the actions of the competition to oppose the most effective actions.

[1] It is a Latin phrase that literally means "other things being the same", and is paraphrased in Spanish translation as "the rest remaining constant". In scientific language, this is the name of the method in which all the variables of a situation are kept constant, except for the one whose influence you want to study. This allows the analysis to be simplified, since otherwise it would be very difficult or impossible to elucidate the effect of each individual variable. If the method is repeatedly applied, orderly varying each of the variables and only one variable at a time, it is possible to understand very complicated phenomena. This method allows the analysis of complex phenomena and facilitates their description.

There is a certain amount of payouts in each game that can be loss for a player and gain for his opponent. This amount is called the "value of the game" (v), and you cannot get more advantage than (v) as a loss or gain. If it is a positive amount, it is precisely what player or company A should pay B on each move, making a fair play. In other games, usually, v = 0.

In a 2 person game, the game rule is usually summarized by a table or “game matrix” that expresses the profit of the maximizing player A or the losses of the minimizing player B if it is a zero-sum bipersonal game, that is, if the gains of one are equal to the losses of the other (Desbazeille, 1969).

The above rectangular matrix can be represented like this:

B (minimizing)

$$\text{A (maximizing)} \quad \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{bmatrix}$$

In a game involving a single move, the choice of a single row of the matrix is called “pure strategy”; if several moves are made, the choice of a certain number of rows of the matrix according to appropriate frequencies will be called “mixed strategy” of A, although this last case is not considered in the example that follows. In any case, a pure or mixed strategy cannot be elaborated until after having chosen a criterion taking into account the attitude of player A.

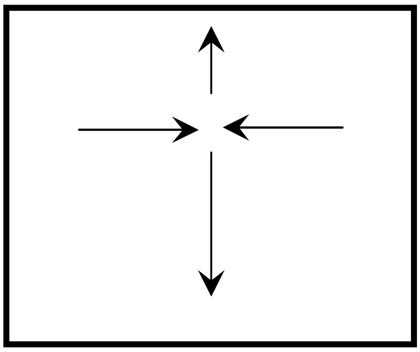
If we consider a Neumannian behavior and consider the matrix $M = [a_{ij}]$, it is shown that, for any matrix M, we have:

$$\underset{j}{\text{MIN}}\,[\underset{i}{\text{MAX}}\; a_{ij}] \geq \underset{i}{\text{MAX}}\,[\underset{j}{\text{MIN}}\; a_{ij}] \,.$$

When the matrix M is such that: $\underset{i}{\text{MAX}}\,[\underset{j}{\text{MIN}}\; a_{ij}] = \underset{j}{\text{MIN}}\,[\underset{i}{\text{MAX}}\; a_{ij}] = v$,

the rectangular set is said to have a *balance point* or *saddle point*.

The saddle point is the one that represents the smallest number in its row and the largest in its column, as can be seen in the following diagram:



Notes: Keep in mind that: 1) A matrix can have different saddle points. 2) A matrix may not have a saddle point (Desbazeille, 1969).

3. EXAMPLE OF APPLICATION

Let's see it through an example.

Let's imagine that a specific work or service is put up for auction with a material execution budget (before taxes and industrial benefit) of:

P = € 42,546,270.00.

Company A, who has thoroughly studied the corresponding technical project and the accompanying administrative documentation, concludes that the cost price of the work will result in an overall amount of C = € 35,500,000.00 (foreseeing, even, the increase in the cost of materials and labor during the execution period, in the event that a price revision is not foreseen). That is: if you intend to compete for its cost, you must cancel the auction from:

$$\mathrm{R} = \frac{42\,546\,270.00 - 35\,500\,000.00}{42\,546\,270.00} \times 100 = 16.561\% .$$

However, how to take into account the reduction that the remaining four competitors can offer? Well, given the experience gained in similar cases, A's management decides to follow any one of these 4 policies (with percentages close to that calculated, somewhat more by default than by excess, for greater security):

- Offer a discount percentage of 10%.
- Offer a discount percentage of 12%.
- Offer a discount percentage of 16%.
- Offer a discount percentage of 20%.

However, A's management does not know "a priori" which of these 4 policies is preferable; the only thing he knows is the price of the budget (P euros), and that A is able to build the work or provide the service for C euros.

If any of the other companies offers a reduction R higher than the reduction offered by A, it will not be awarded the work and, therefore, will not be able to obtain any financial gain. On the contrary, if the discount offered by A is rationally superior to any of the other discounts, the work will be awarded (apart from other legal and administrative conditions) and, with it, some possible benefits.

In this situation, A's "profit matrix" is obviously the following:

	Maximum rebate of A's competitors				
Rebate of A	**R<10%**	**10%≤R<12%**	**12%≤R<16%**	**16%≤R<20%**	**R ≥20%**
10%	0.90 P-C	0	0	0	0
12%	0.88 P-C	0.88 P-C	0	0	0
16%	0.84 P-C	0.84 P-C	0.84 P-C	0	0
20%	0.80 P-C	0.80 P-C	0.80 P-C	0.80 P-C	0

Table 1. Game matrix.

For example, if A lowers 16% and B, which is the second-highest concurrent company, drops 14%, the work is awarded to A, which will charge a price of:

$$P - 0.16 \cdot P = (1 - 0.16)\cdot P = 0.84\cdot P ,$$

then the gain of A is equal to: **0.84 P-C**, value that has been written in the box corresponding to the "third row - third column" of the previous matrix. The values in the other cells are derived in the same way. In addition, applying the criterion of prudence, A's management admits that in the event of a tie down, the work will be awarded, by the competent body, to the other company that reduced the same percentage as A.

4. CRITERIA USING STATES OF NATURE

4.1. Previous considerations

Applying Wald's pessimistic criteria, which will be seen later, the person who makes the decision thinks that once a certain strategy has been selected, the most unfavourable state of nature will be presented and he will choose the strategy that gives him the most favourable remuneration among the worst. It would be a situation of uncertainty in which the states of nature are not used.

However, on the contrary, here we are in a "risk" situation, in which we know the list of states of nature as well as their probability of occurrence. Since the events are mutually exclusive[2], it turns out that the sum of the probabilities of all the states must be equal to unity (total probability). Then, the "mathematical expectation" or average value is calculated and the largest one is chosen for each row (Franquet, 2012).

4.2. Laplace equiprobability criterion

The different possible states of nature, having unknown probabilities, are considered as equiprobable. That is, in the face of ignorance of the probabilities of each case, the decision maker considers that the different states of nature have the same probability. It is, therefore, the only criterion that shows indifference, since it faces uncertainty[3], granting an equiprobability (or equal probability) of occurrence to all possible states of nature. This criterion, proposed by Laplace in 1825, is based on the principle of "insufficient reason". Since the probabilities associated with the occurrence are unknown, there is insufficient information to conclude that these probabilities will be different. Therefore, due to insufficient reason to believe otherwise, states of nature all have the same possibility of occurring. When this conclusion is established, the problem becomes a decision with "risk", where the action that provides the highest expected profit is chosen.

Fig. 1. Pierre-Simon Laplace (1749-1827).

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[2] In probability theory, events or events E_1, E_2, ..., E_n are said to be **mutually exclusive** if the occurrence of one of them implies the non-occurrence of the other n - 1 events. Therefore, two mutually exclusive events cannot occur simultaneously. In formal language, the intersection of each pair of them is the empty set (the null event): $A \cap B = \emptyset$. Therefore, mutually exclusive events have the property that: $P(A \cap B) = 0$.

[3] *Uncertainty* is "imperfection in knowledge about the state or processes of nature" (FAO/Government of Sweden, 1995). Statistical uncertainty is "randomness or error from various sources such as those described when using statistical methodology." When management decisions are to be based on quantitative estimates from evaluation models, it is desirable that the uncertainty be quantified and used to calculate the probability of achieving the desired objective and/or incurring undesirable events.

A's management is neither pessimistic nor optimistic. It simply lacks information on the offers of its competitors (probably, it does not even know who and how many they are) and it does not have any evidence to suppose that "nature" (other companies) will play a better role than any other. . In short, it ignores the possibilities that the maximum discount offered by A's 4 competitors is located in a certain column of the five possible ones in the previous profit matrix; nor does he dare to speak in favor or against any possibility. Then, its absolute neutrality inclines it to grant equal possibilities to each and every one of the columns of the matrix (that is, a p = 20% $= \frac{1}{5}$), so it adopts the Laplacian[4] hypothesis of equiprobability of the columns that represent all the states of nature (e_j).

Under these conditions, the player will choose the row corresponding to:

$$\underset{i}{\mathrm{MAX}}\left[\frac{a_{i1} + a_{i2} + ... + a_{in}}{n}\right],$$

that is, the row for which the mathematical expectation or average value of your earnings is largest. It is seen that the game matrix is simply reduced to a column matrix. If the probabilities of the different possible states of nature are known, the criterion of maximum mathematical expectation will also be used, reducing the game matrix to a column matrix.

If we now define the "mathematical hope"[5] of a game as the product of the prize (or punishment) that it offers for the probability of obtaining it, we see that the following cases can occur, taking into account that:

$$E_i = p_j \times a_{ij} \ (\forall i \in [1,4];\ \forall j \in [1,5]).$$

1) If A offers a 10% discount, his mathematical expectation of profit is:

$$E_1 = \frac{1}{5}(0.90\,P - C) + \frac{1}{5}\times 0 + \frac{1}{5}\times 0 + \frac{1}{5}\times 0 + \frac{1}{5}\times 0 = \frac{1}{5}(0.90\,P - C);$$

[4] **Pierre-Simon Laplace** was a French astronomer, physicist, and mathematician. Continuer of Newtonian mechanics, he discovered and developed the Laplace transform (very useful for solving differential and integral equations) and the Laplace equation; As a statistician, he laid the foundations for the analytical theory of probability.

[5] In Statistics, the **mathematical expectation** (also called **hope**, **expected value**, **population mean**, or simply **mean**) of a random variable is the number that formalizes the idea of the *mean value* of a random phenomenon. When the random variable is discrete, the mathematical expectation is equal to the sum of the probability of each possible random event multiplied by the value of that event. Therefore, it represents the average amount that is "expected" as a result of a random experiment when the probability of each event remains constant and the experiment is repeated a high number of times. It should be said that the value that mathematical hope takes in some cases may not be "expected" in the most general sense of the word (the value of hope may be improbable or even impossible).

2) If A offers a 12% discount, his mathematical expectation of profit is:

$$E_2 = \frac{1}{5}(0.88\,P - C) + \frac{1}{5}(0.88\,P - C) + \frac{1}{5}\times 0 + \frac{1}{5}\times 0 + \frac{1}{5}\times 0 = \frac{2}{5}(0.88\,P - C);$$

3) If A offers a 16% discount, his mathematical expectation of profit is:

$$E_3 = \frac{1}{5}(0.84\,P - C) + \frac{1}{5}(0.84\,P - C) + \frac{1}{5}(0.84\,P - C) + \frac{1}{5}\times 0 + \frac{1}{5}\times 0 = \frac{3}{5}(0.84\,P - C);$$

4) If A offers a 20% discount, his mathematical expectation of profit is:

$$E_4 = \frac{1}{5}(0.80\,P - C) + \frac{1}{5}(0.80\,P - C) + \frac{1}{5}(0.80\,P - C) + \frac{1}{5}(0.80\,P - C) + \frac{1}{5}\times 0 = \frac{4}{5}(0.80\,P - C),$$

since the hypothesis of equal probability (p = 1/5 = 20%) is admitted for each column of the previous matrix.

Well, since we know that P = 42, 546, 270.00 €, we can draw, in Fig. 2, the linear (straight) functions obtained from the mathematical expectations based on the cost of the work, which will be, respectively:

$$\begin{cases} E_1 = \frac{1}{5}(38\,291\,643.00 - C) \\ E_2 = \frac{2}{5}(37\,440\,718.00 - C) \\ E_3 = \frac{3}{5}(35\,738\,867.00 - C) \\ E_4 = \frac{4}{5}(34\,037\,016.00 - C) \end{cases}$$

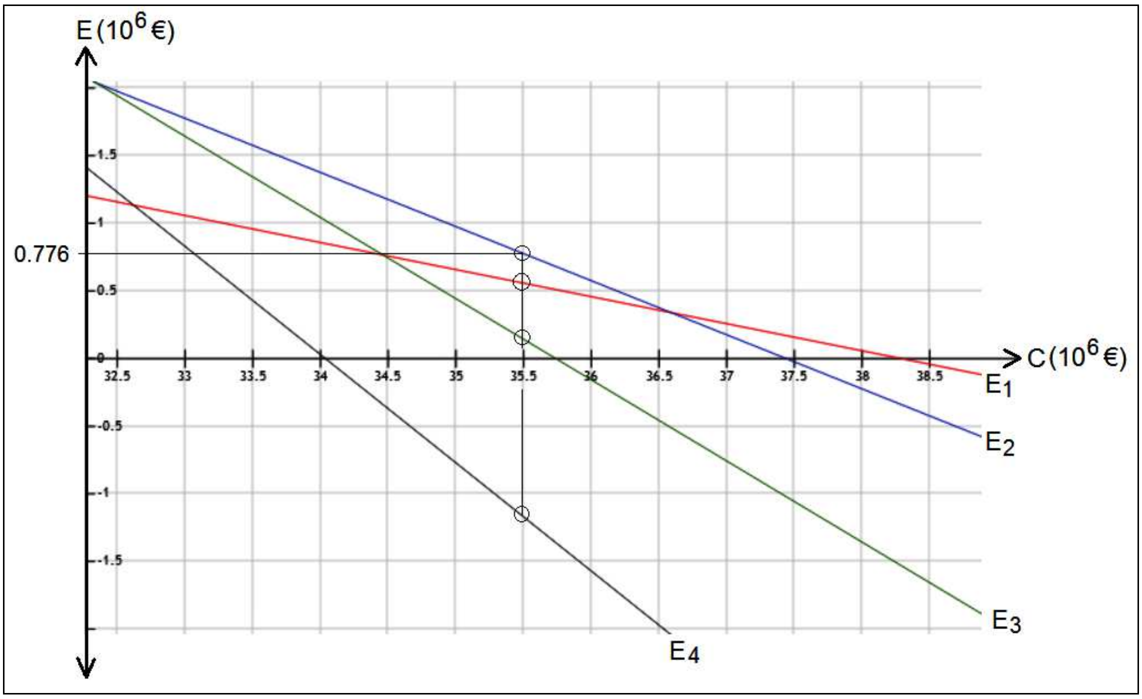

Fig. 2. Various mathematical hopes (I).

Substituting the estimated value of C in the previous expressions, we have to:

$$\begin{cases} E_1 = \frac{1}{5}(38\,291\,643.00 - 35\,500\,000.00) = 558\,328.60\,€ \\ \mathbf{E_2 = \frac{2}{5}(37\,440\,718.00 - 35\,500\,000.00) = 776\,287.20\,€} \\ E_3 = \frac{3}{5}(35\,738\,867.00 - 35\,500\,000.00) = 143\,320.20\,€ \\ E_4 = \frac{4}{5}(34\,037\,016.00 - 35\,500\,000.00) = -1\,170\,387.20\,€ \end{cases}$$

, with E_2 being the highest mathematical expectation obtained, and E_4 even being a negative mathematical hope[6].

This can also be seen in Fig. 2 since, in our case, the expected cost of carrying out the work, as we have seen, the amount of which is C = € 35,500,000, corresponds to a mathematical expectation in the graph. maximum of type $\underline{E_2}$, so **company A must offer a 12% drop in the auction**.

[6] Whenever we choose a certain system we have to check that it has positive mathematical hope, since if a system or strategy has negative mathematical hope it is a losing system and, in the long term, it will make us lose money.

Or what is the same: being the expected cost of € 35,500,000.00, **there is the same probability of winning the work by going down 16.561% in the auction than by going down 12% (although it seems paradoxical), which is why and with the In order to be discounted the minimum, A must lower only 12%**.

In general, the following table-summary of the various losses in the auction that can be offered by A can be prepared, according to the expected costs C of the work, deducting the extremes of the class intervals, namely:

EXPECTED COST OF WORK (€)	**POLICY TO FOLLOW**	**LOW AT AUCTION**
C < 28,931,436.60	E_4	20%
28,931,436.60 ≤ C < 32,335,165.20	E_3	16%
32,335,165.20 ≤ C < 36,589,729.20	E_2	12%
36,589,729.20 ≤ C < 38,291,643.00	E_1	10%
C ≥ 38,291,643.00	"according to cost" (normal method)	$\frac{P-C}{P}\times 100$

Table 2. Losses in the auction according to cost intervals.

4.3. Criterion of expected profit or subjective probabilities

It may be the case that A's management, due to the good information it has about the other bidding companies, does not grant equal probability to the five columns of the matrix in question. In this case, the probabilities of occurrence of the various states of nature are different.

Here two different assumptions have been made (although we could have considered some more), for each of which the corresponding expected profit or expected value will be calculated. So:

a) **None of the competing companies usually risk at auctions, not usually falling more than 10%**. In this case, you can give the 1st column of the matrix a probability of p_1 = 3/5 = 60%, the 2nd column a probability of p_2 = 1/5= = 20% and the other columns of the matrix a probability of

$p_3 = p_4 = p_5$ = 1/15 = 6.67%, so that a total probability of:

$$\frac{3}{5}+\frac{1}{5}+\frac{1}{15}+\frac{1}{15}+\frac{1}{15}=1,$$

and the following mathematical expectations are obtained, considering the same assumptions as in the previous case:

$$
\begin{cases}
E_1 = \frac{3}{5}(0.90\,P - C) + \frac{1}{5}\times 0 + \frac{1}{15}\times 0 + \frac{1}{15}\times 0 + \frac{1}{15}\times 0 = \frac{3}{5}(0.90\,P - C) \\
E_2 = \frac{3}{5}(0.88\,P - C) + \frac{1}{5}(0.88\,P - C) + \frac{1}{15}\times 0 + \frac{1}{15}\times 0 + \frac{1}{15}\times 0 = \frac{4}{5}(0.88\,P - C) \\
E_3 = \frac{3}{5}(0.84\,P - C) + \frac{1}{5}(0.84\,P - C) + \frac{1}{15}(0.84\,P - C) + \frac{1}{15}\times 0 + \frac{1}{15}\times 0 = \frac{13}{15}(0.84\,P - C) \\
E_4 = \frac{3}{5}(0.80\,P - C) + \frac{1}{5}(0.80\,P - C) + \frac{1}{15}(0.80\,P - C) + \frac{1}{15}(0.80\,P - C) + \frac{1}{15}\times 0 = \frac{14}{15}(0.80\,P - C)
\end{cases}
$$

with what, probably, the result will be different from what we would obtain applying the antecedent criterion of the Laplacian equiprobability. For its calculation, we would follow the same process as the one previously indicated, and then:

$$
\begin{cases}
\mathbf{E_1 = \frac{3}{5}(0.90\times 42\,546\,270.00 - 35\,500\,000.00) = 1\,674\,985.80\ €} \\
E_2 = \frac{4}{5}(0.88\times 42\,546\,270.00 - 35\,500\,000.00) = 1\,552\,574.10\ € \\
E_3 = \frac{13}{15}(0.84\times 42\,546\,270.00 - 35\,500\,000.00) = 207\,017.89\ € \\
E_4 = \frac{14}{15}(0.80\times 42\,546\,270.00 - 35\,500\,000.00) = -1\,365\,451.70\ €
\end{cases}
$$

As can be seen, in this case it would be convenient to choose the E_1 strategy which, in the following Fig. 3, reaches the maximum value of the mathematical expectation for the cost of € 35,500,000.00.

This criterion involves selecting the alternative whose expected or average payment is better (if the payments are benefits, the one with the highest expected benefit, and if they are costs, the one with the lowest expected cost). This criterion is the most common when the probabilities are known, but it does not have to be the most appropriate. Note that if the decision process is repeated many times in identical conditions, the laws of large numbers ensure that in the limit the average payment is hope (Vitoriano, 2017).

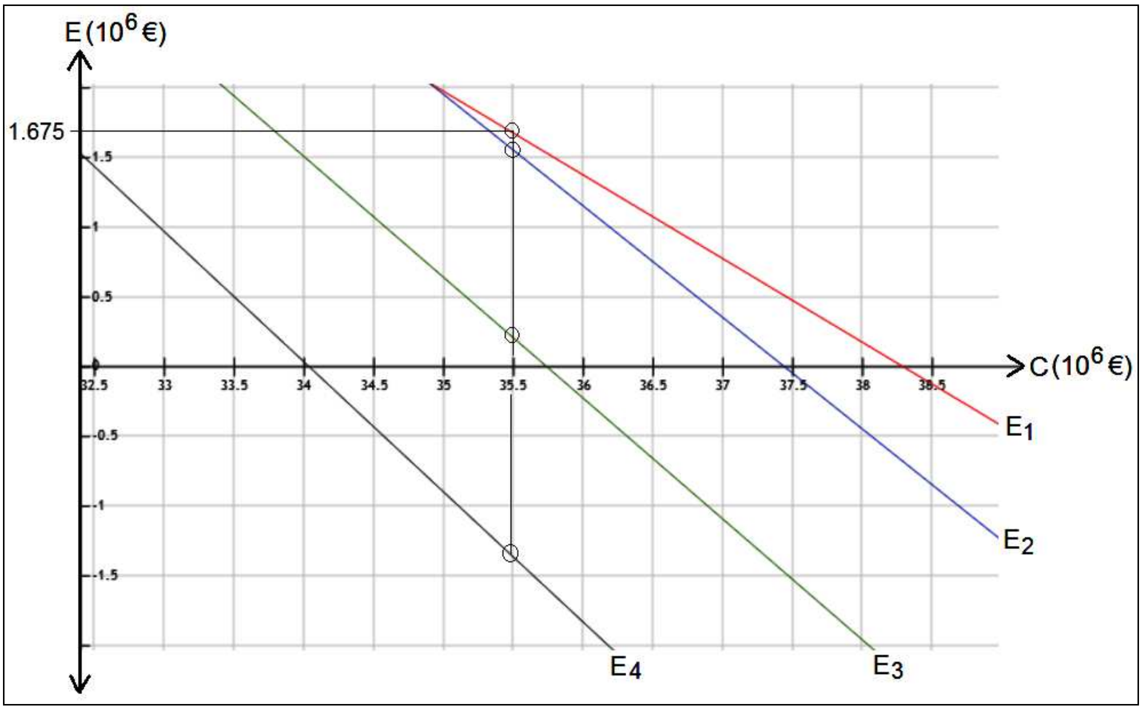

Fig. 3. Various mathematical hopes (II).

b) **One of the companies that attends the auction has the "irrational" custom, be it the work or service as it may be, of going down by system in its offer more than 20%.**

In this case, we will grant the 5th column of the matrix, v. gr., a probability of $p_5 = 4/5 = 80\%$, and the remaining four columns, a probability of $p_1 = p_2 = p_3 = p_4 = 1/20 = 5\%$ each, so that a total probability will result of:

$\frac{1}{20}+\frac{1}{20}+\frac{1}{20}+\frac{1}{20}+\frac{4}{5}=1$. Then, we would be left with:

$$\begin{cases} E_1 = \frac{1}{20}(0.90\,P - C) + \frac{1}{20}\times 0 + \frac{1}{20}\times 0 + \frac{1}{20}\times 0 + \frac{4}{5}\times 0 = \frac{1}{20}(0.90\,P - C) \\ E_2 = \frac{1}{20}(0.88\,P - C) + \frac{1}{20}(0.88\,P - C) + \frac{1}{20}\times 0 + \frac{1}{20}\times 0 + \frac{4}{5}\times 0 = \frac{1}{10}(0.88\,P - C) \\ E_3 = \frac{1}{20}(0.84\,P - C) + \frac{1}{20}(0.84\,P - C) + \frac{1}{20}(0.84\,P - C) + \frac{1}{20}\times 0 + \frac{4}{5}\times 0 = \frac{3}{20}(0.84\,P - C) \\ E_4 = \frac{1}{20}(0.80P - C) + \frac{1}{20}(0.80P - C) + \frac{1}{20}(0.80P - C) + \frac{1}{20}(0.80P - C) + \frac{4}{5}\times 0 = \frac{1}{5}(0.80P - C) \end{cases}$$

Also, here, the result (the choice of the percentage of loss in the auction) may be different from the one we have previously calculated, and the alternative E_2 must be chosen because it is the highest, as has happened in the first assumption. Indeed:

$$
\begin{cases}
E_1 = \dfrac{1}{20}(0.90 \times 42\,546\,270.00 - 35\,500\,000.00) = 139\,582.15 \text{ €} \\
\mathbf{E_2 = \dfrac{1}{10}(0.88 \times 42\,546\,270.00 - 35\,500\,000.00) = 194\,071.76 \text{ €}} \\
E_3 = \dfrac{3}{20}(0.84 \times 42\,546\,270.00 - 35\,500\,000.00) = 35\,830.02 \text{ €} \\
E_4 = \dfrac{1}{5}(0.80 \times 42\,546\,270.00 - 35\,500\,000.00) = -292\,596.80 \text{ €}
\end{cases}
$$

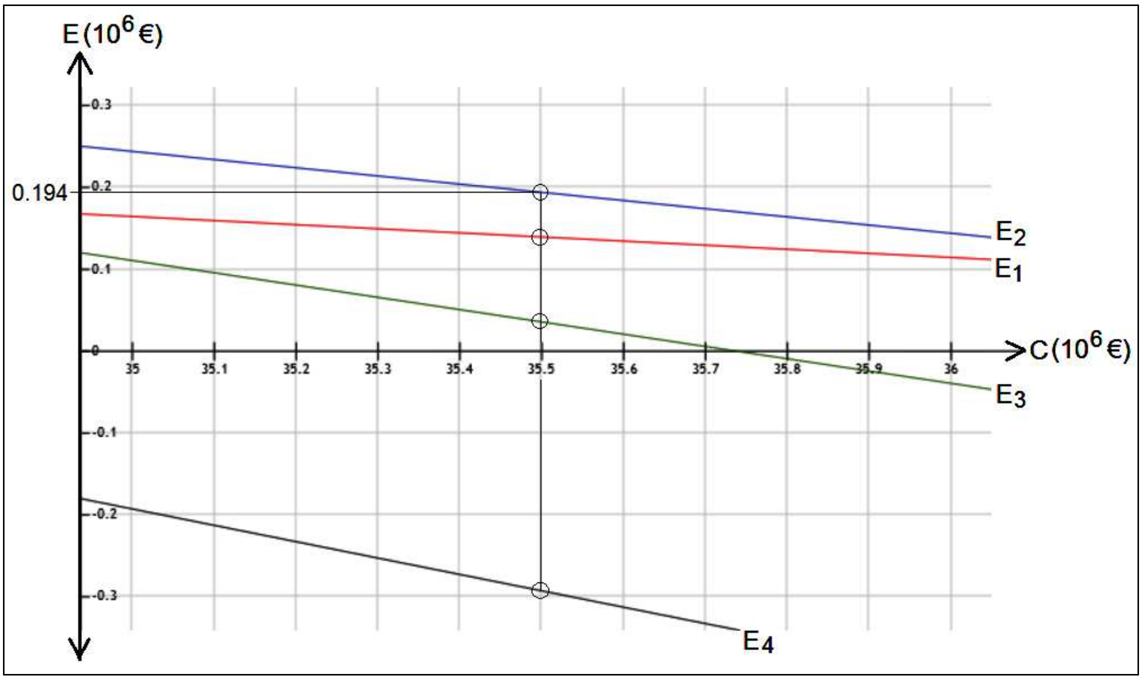

Fig. 4. Various mathematical hopes (III).

4.4. Most likely criteria

The most probable state of nature will be chosen and, for that state, the best alternative or strategy. In our case, we will have to consider the assumptions a) and b) of the previous heading (expected profit). If we now substitute the corresponding monetary values of the game matrix, whose elements are the different a_{ij}, we will obtain:

N

↓ (e1) ... ↓ (e5)

A		e_1	e_2	e_3	e_4	e_5
		p_1	p_2	p_3	p_4	p_5
	E_1	2 791 643	0	0	0	0
	E_2	1 940 718	1 940 718	0	0	0
	E_3	238 867	238 867	238 867	0	0
	E_4	-1 462 984	-1 462 984	-1 462 984	-1 462 984	0

Case a) The most probable state of nature is e_1, with $p_1 = 3/5 = 0.6$, and for that state the best alternative is E_1 (€ 2,791,643).

Case b) The most probable state of nature is e_5, with $p_5 = 4/5 = 0.8$, and for that state any alternative (€ 0) is valid.

4.5. Middle scenario criteria

The mean scenario (MS) is calculated by weighting the probabilities of occurrence of each state of nature, choosing the alternative corresponding to the highest gain obtainable, which must frequently be found by interpolation between two consecutive values. In other words, in the case of Laplacian equiprobability, we would have:

$$MS = 1\times0.2 + 2\times0.2 + 3\times0.2 + 4\times0.2 + 5\times0.2 = 0.2(1 + 2 + 3 + 4 + 5) = 3,$$

which corresponds to the state of nature e_3. The earnings would be as follows:
0 € for E_1; 0 € for E_2; 238,867 € for E_3; -1,462,984 € for E_4.

So the chosen alternative would be E_3.

If both assumptions of the two previous headings had been considered, one would have:

Case a):

$$MS = 1\times3/5 + 2\times1/5 + 3\times1/15 + 4\times1/15 + 5\times1/15 = 9/5 = 1.8,$$

that does not correspond exactly to any of the states of nature since $MS \notin \{N\}$, although if we proceed to estimate by linear interpolation between states 1 and 2, the higher a_{ij} = € 1,940,718 results, so we will choose the E_2 alternative.

Case b):

$$MS = 1\times1/20 + 2\times1/20 + 3\times1/20 + 4\times1/20 + 5\times4/5 = 9/2 = 4.5,$$

which does not correspond to any of the states either since $MS \notin \{N\}$, although if we proceed by interpolation between 4 and 5 a negative profit (loss) of:
a_{ij} = -1,462,984 €/2 = -731,492 €, by which is counterproductive the assumption of any alternative.

4.6. Criterion of variability of results

The fact of using mathematical hope as the only decision criterion implies also assuming certain starting hypotheses that may seem questionable, namely:

- That the decision-maker does not care about the dispersion or variability of the result (the standard deviation, the mean deviation, the variance, the range or any other measure of the absolute or relative dispersion of the sample are not taken into account).

- That there is no risk of ruin or bankruptcy: it is the risk that the outcome of a certain strategy may suppose an economic loss such that it cannot be overcome by the company. In this case, the decision maker would choose among the alternatives, the most unfavourable results that can be assumed by company A. This has to do with the ability to assume losses.

Well, to solve these annoying limitations, certain utility functions are built, as will be seen below.

Considering the variability of the results implies penalizing economic hope with a measure that provides an idea about the variability of the data (earnings), considering the multiplication of this measure of variability by a certain *coefficient indicative of fear of risk* (β), such that $0 \leq \beta \leq 1$, of the decision maker (company A). The utility function is found by subtracting the aversion coefficient multiplied by the sample standard deviation σ_{n-1} from the expected value of each alternative and considering the following extreme values:

- If $\beta \rightarrow 1$, greater risk aversion. The decision maker presents a more typically conservative profile.

- If $\beta \rightarrow 0$, little risk aversion. The decision-maker presents a riskier profile.

$$\textit{Utility function} = U(E_i) = E_i - \beta \cdot \sigma_i \ .$$

Note that when β tends to 1, the quantity to be subtracted is greater, therefore the expected utility is also less, which corresponds to a conservative profile. On the other hand, the standard or "typical" deviations of the sample of the five winnings of each row of the game matrix are the following:

σ_1 = 1,248,460.70 €
σ_2 = 1,062,975.00 €
σ_3 = 130,832.84 €
σ_4 = 654,266.33 €

In our case, there are five states of nature, eg with their respective probabilities of occurrence different according to the three assumptions previously analyzed, and a risk aversion by A's managers of 20% is assumed, which will mean: for each assumption, the following determinations:

Laplace:

$$\begin{cases} U(E_1) = E_1 - \beta \cdot \sigma_1 = 558\,328.60 - 0.2 \times 1\,248\,460.70 = 308\,636.46\ € \\ \mathbf{U(E_2) = E_2 - \beta \cdot \sigma_2 = 776\,287.20 - 0.2 \times 1\,062\,975.00 = 563\,692.20\ €} \\ U(E_3) = E_3 - \beta \cdot \sigma_3 = 143\,320.20 - 0.2 \times 130\,832.84 = 117\,153.63\ € \\ U(E_4) = E_4 - \beta \cdot \sigma_4 = -1\,170\,387.20 - 0.2 \times 654\,266.33 = -1\,301\,240.50\ € \end{cases}$$

Thus, the E_2 alternative would be chosen because it is the largest of all.

Case a):

$$\begin{cases} \mathbf{U(E_1) = E_1 - \beta \cdot \sigma_1 = 1\,674\,985.80 - 0.2 \times 1\,248\,460.70 = 1\,425\,293.70\ €} \\ U(E_2) = E_2 - \beta \cdot \sigma_2 = 1\,552\,574.10 - 0.2 \times 1\,062\,975.00 = 1\,339\,979.10\ € \\ U(E_3) = E_3 - \beta \cdot \sigma_3 = 207\,017.89 - 0.2 \times 130\,832.84 = 180\,851.32\ € \\ U(E_4) = E_4 - \beta \cdot \sigma_4 = -1\,365\,451.70 - 0.2 \times 654\,266.33 = -1\,496\,305.00\ € \end{cases}$$

Thus, the E_1 alternative would be chosen because it is the largest of all.

Case b):

$$\begin{cases} U(E_1) = E_1 - \beta \cdot \sigma_1 = 139\,582.15 - 0.2 \times 1\,248\,460.70 = -110\,109.99\ € \\ U(E_2) = E_2 - \beta \cdot \sigma_2 = 194\,071.76 - 0.2 \times 1\,062\,975.00 = -18\,523.24\ € \\ \mathbf{U(E_3) = E_3 - \beta \cdot \sigma_3 = 35\,830.02 - 0.2 \times 130\,832.84 = 9663.45\ €} \\ U(E_4) = E_4 - \beta \cdot \sigma_4 = -292\,596.80 - 0.2 \times 654\,266.33 = -423\,450.07\ € \end{cases}$$

Here the E_3 alternative would be chosen because it is the largest of them all and, by the way, the only positive one.

Finally, it must be taken into account that both the values of the mathematical expectation (and, consequently, of the probabilities assigned to each state of nature) and the parameter β decisively influence the computation of the corresponding expected utility, up to the point to be able to vary the strategy initially adopted.

Thus, for example, if we consider the $U(E_1)$ chosen from case a) and study the variability of the results, considering 4 different values that the decision-maker can adopt in relation to the expressed parameter (20%, 40%, 60% and 80%, saving its extreme values), we can represent the following Fig. 5, in which:

$$\begin{cases} U(E_1) = E_1 - \beta_1 \cdot \sigma_1 = 1\,674\,985.80 - 0.2 \times 1\,248\,460.70 = 1\,425\,293.70\ € \\ U(E_1) = E_1 - \beta_2 \cdot \sigma_1 = 1\,674\,985.80 - 0.4 \times 1\,248\,460.70 = 1\,175\,601.52\ € \\ U(E_1) = E_1 - \beta_3 \cdot \sigma_1 = 1\,674\,985.80 - 0.6 \times 1\,248\,460.70 = 925\,909.38\ € \\ U(E_1) = E_1 - \beta_4 \cdot \sigma_1 = 1\,674\,985.80 - 0.8 \times 1\,248\,460.70 = 676\,217.24\ € \end{cases}$$

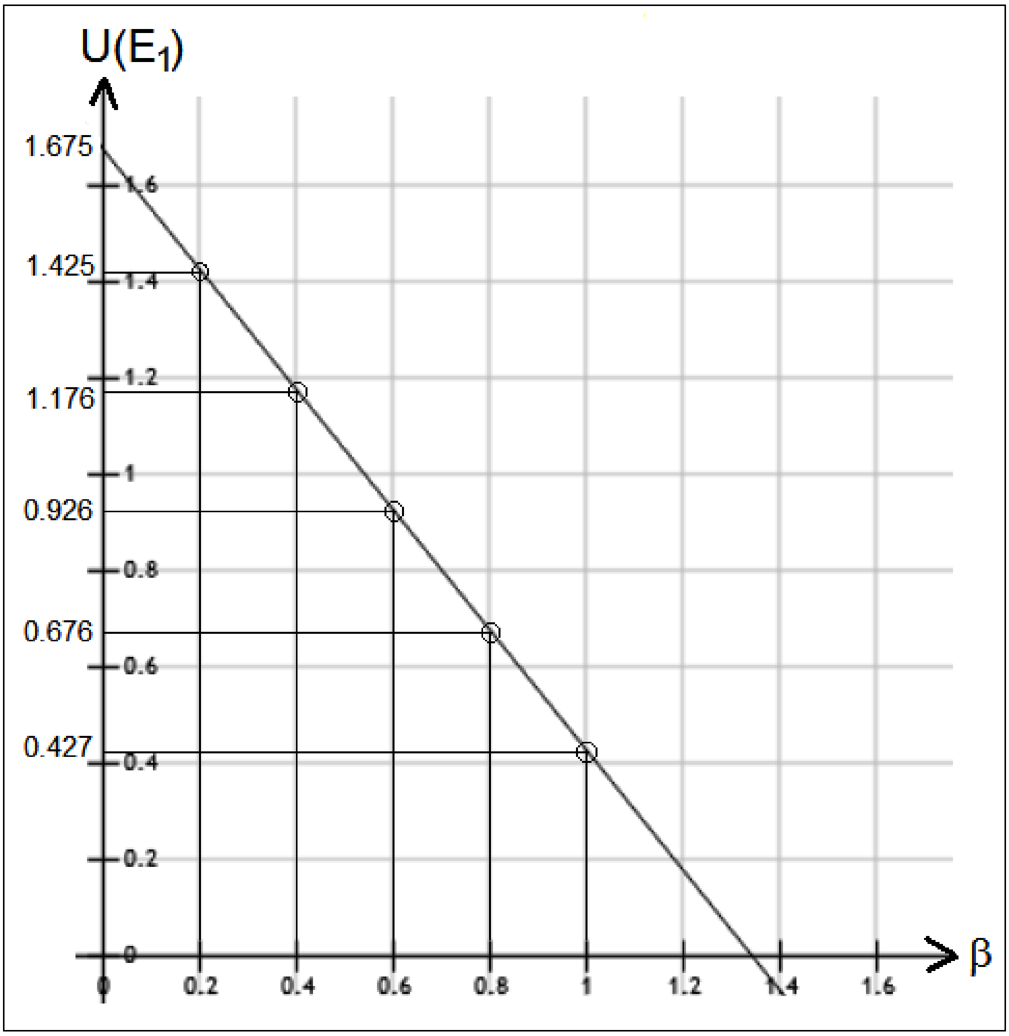

Fig. 5. Function U = f(β).

5. CRITERIA WITHOUT USING THE STATES OF NATURE

5.1. Previous considerations

The truth is that the strategy to be followed cannot be elaborated until after a certain criterion has been chosen, taking into account the attitude of company A. Until now we have applied the Laplace criterion or the “insufficient reasoning”, according to which all the probabilities of occurrence of the states of nature are the same, or other assumptions in which said probabilities could be different. These are situations of "uncertainty". Next, other commonly used criteria to solve this type of decision problem will be applied to the previous example, which do not use the probabilities of the states of nature, and which are used when those probabilities are unknown or it is preferred that they be ignored.

5.2. Von Neumann's criterion or Wald's (pessimistic) criterion

This "minimax" theorem dates from 1928, initially due to Von Neumann and later perfected (Wald, 1945). It states that in certain zero-sum games (zero-sum two-person games), which involve perfect information (that is, that each player knows in advance the strategy of their opponent and its consequences), there is a strategy that allows both players to minimize their maximum loss (hence the name "minimax"). In particular, when examining each possible strategy, a player must consider all the possible responses of the opposing player and the maximum loss that this may entail. The player then plays with the strategy that results in minimizing his maximum loss. Such a strategy is considered optimal for both players only if their minimaxes are equal (in absolute value) and opposite (in sign). If the common value is zero, the game becomes nonsense.

Companies A and B are supposed to act smart and prudent. Under these conditions, A will choose the row of the matrix in which its smallest profit is maximum, while B or N will choose among all the columns the one in which its greatest loss is minimum. This behavior corresponds to the attitude of a company that does not want to take any risk[7].

Thus, A will choose the row corresponding to:

$$\underset{i}{\text{MAX}}\,[\underset{j}{\text{MIN}}\,a_{ij}] \quad \begin{cases} \forall i = 1, 2, ..., m, \\ \forall j = 1, 2, ..., n. \end{cases}$$

and B the column corresponding to:

$$\underset{j}{\text{MIN}}\,[\underset{i}{\text{MAX}}\,a_{ij}] \quad \begin{cases} \forall i = 1, 2, ..., m, \\ \forall j = 1, 2, ..., n. \end{cases}$$

[7] From an economic point of view, risk means uncertainty about the evolution of an asset, and indicates the possibility that an investment offers a different return than expected (both in favor and against the investor, although logically the latter is only concerned with the risk of recording losses). It can also be understood as a measure of the extent of the damage in the face of a dangerous situation. The risk is measured assuming a certain vulnerability against each type of danger. Although it is not always done, an adequate distinction must be made between dangerousness (probability of occurrence of a hazard), vulnerability (probability of occurrence of damage given that a hazard has occurred) and risk (proper).

Fig. 6. John Von Neumann (1903-1957).

For each alternative, it is assumed that the worst will happen, choosing the alternative that offers the best value. In this way, it is ensured that, in the worst case, the best possible is obtained, which corresponds to a pessimistic view of what can happen. In the event that the payments are costs, this approach philosophy involves choosing the minimum of the maximums ("minimax"), while if they are profits, it will be the maximum of the minimums ("maximin") (Vitoriano, 2007).

The matrix of the game we have seen is:

		N							
	E_1	2 791 643	0	0	**0**	**0**	0	←	MAXIMIN
	E_2	1 940 718	1 940 718	0	**0**	**0**	0	←	
A	**E_3**	238 867	238 867	238 867	**0**	**0**	0	←	
	E_4	-1 462 984	-1 462 984	-1 462 984	-1 462 984	0	-1 462 984		
		2 791 643	1 940 718	238 867	0	0			
					↑	↑			
				MINIMAX					

The pure strategy to be followed by company A would be given by any of the first three rows (E_1, E_2, E_3). If nature were replaced by an intelligent and prudent player, he would choose columns 4 and 5. In short, there are six balance points or saddle points, of value: v = € 0.

5.3. Optimistic criteria

This criterion is based on the choice of the best possible case. Consider optimistic and aggressive views. An optimistic decision maker believes that you will always get the best result regardless of the decision made. An aggressive decision maker chooses the decision that will give him the most profit. To find the optimal decision, the maximum profit is marked for each of the decision alternatives and the decision that has the maximum profit is selected. This criterion is the only one that is based on the principle that, once the decision has been made, nature always favors this decision, showing itself to be absolutely optimistic and happy for what is won. This involves some drawbacks on which we will not extend here for obvious reasons of space.

It is the "maximax" criterion, just opposite to the previous one. For each alternative it is assumed that the best will happen, choosing the one that offers the best value. This criterion is hardly used since it does not take into account, at any time, the risks that are taken when making a certain decision (Vitoriano, 2007).

		N						MAXIMAX
A	**E_1**	2 791 643	0	0	0	0	2 791 643	←
	E_2	1 940 718	1 940 718	0	0	0	1 940 718	
	E_3	238 867	238 867	238 867	0	0	238 867	
	E_4	-1 462 984	-1 462 984	-1 462 984	-1 462 984	0	-1 462 984	

In this case, the maximums of the rows coincide with the profits of each one of them, so the pure strategy E_1 would be chosen.

5.4. Hurwicz criterion or partial optimism

This criterion (Hurwicz, 1945) Solomonically combines pessimistic and optimistic attitudes, valuing each alternative with a weight between the best and the worst possible. The weighting is carried out by multiplying the best by a certain factor α between 0 and 1, called the "optimism index", and the worst by $(1 - \alpha)$, adding both quantities. The alternative that offers the best value will be chosen.

The criterion in question presents the difficulty of subjectively estimating the value of the decision maker's optimism index, in such a way that usually the solution is obtained for all the possible values of this index and an attempt is made to place the decision maker in one of the resulting intervals of the index of optimism (Vitoriano, 2007). However, the adoption of an intermediate value how $\alpha = ½$ is quite normal.

Fig. 7. Leonid Hurwicz (1917-2008).

Thus, the optimism of A is defined by the factor α / $0 \leq \alpha \leq 1$. If D and d are, respectively, the largest and smallest of the elements in a row, the row corresponding to:

$$\text{Max } [\alpha \cdot D + (1-\alpha) \cdot d].$$

So, in our case, considering $\alpha = ½$ (50% optimism and 50% pessimism), we have:

Row 1: ½ × 2,791,643 + ½ × 0 = 1,395,821.50 €
Row 2: ½ × 1,940,718 + ½ × 0 = 970,359.00 €
Row 3: ½ × 238,867 + ½ × 0 = 119,433.50 €
Row 4: ½ × 0 + ½ × (-1,462,984) = -731,492.00 €

, then the E_1 strategy will be chosen, even independently of the subjectively adopted value of the parameter α, as can be verified.

When the game matrix only has 2 columns, the Laplace criterion is but a particular case of the Hurwicz[8] criterion for a half optimistic and half pessimistic entrepreneur. But when said matrix has more than two columns, the same thing no longer happens. While Hurwicz's criterion only involves two columns in the evaluation of each strategy to be followed, Laplace's criterion involves all the columns in the evaluation. In general, the results obtained with the application of both criteria will differ for this reason, as in our example (Ballestero, 1973).

[8] **Hurwicz** was a Polish-American economist and mathematician. His academic recognition is due primarily to his research on mechanism design and incentive compatibility theory. Both are widely used in economics, sociology and political science as instruments to achieve the design of institutions that optimize certain given results. He was one of the first economists to recognize the value of Game Theory, and had been a pioneer in its application.

5.5. Savage or Repentance Criterion

This criterion (Savage, 1955) takes into account the opportunity cost[9], penalty or repentance for not correctly foreseeing the state of nature. These opportunity costs are evaluated for each alternative and each state, making the difference between the best of that state and what that alternative provides for that state, constructing the so-called “matrix of penalties or opportunity costs”. The above criteria are applied to this matrix, and the expected cost can be applied, or what is more usual, the minimum criterion, thus also being known as the criterion of minimizing maximum repentance.

This criterion would be used by people who are afraid of making mistakes and subsequently regretting it. A new matrix is built with the opportunity costs based on not choosing the best strategy in each state of nature. Then, the highest values (largest costs) of each strategy are found. Finally, the lowest value that represents the strategy that has the lowest opportunity cost is chosen.

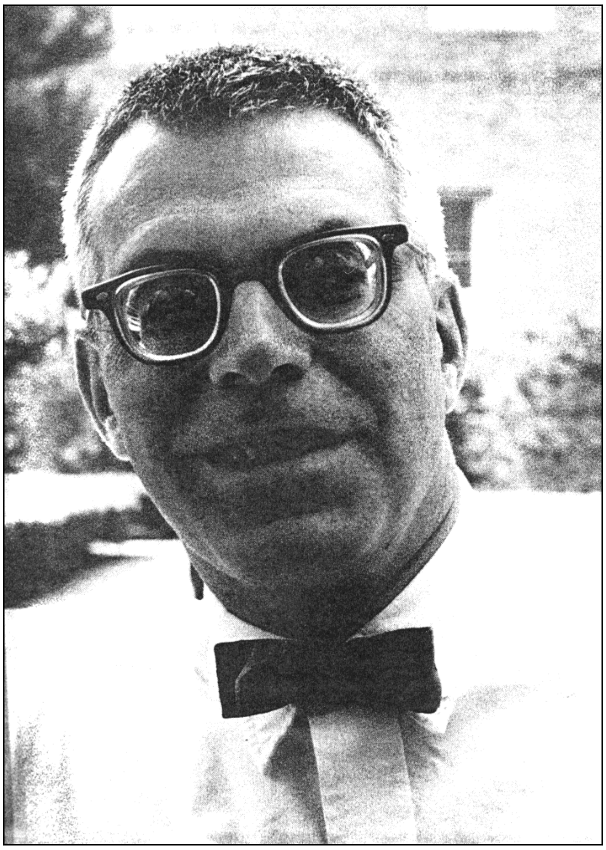

Fig. 8. Leonard Jimmie Savage (1917-1971).

[9] The “opportunity cost” is the cost of the alternative that we give up when we make a certain decision, including the benefits that we could have obtained from choosing the alternative option. Therefore, the opportunity costs are those resources that we no longer perceive or that represent a cost due to the fact that we have not chosen the best possible alternative, when we have limited resources (generally money, work factor and time). The term "opportunity cost" is also referred to as "the value of the best unselected option."

The first step, then, is to build the matrix of penalties, damages ("penalties or regrets") or opportunity costs, whose elements α_{ij} represent the deviation between the realized and the presumed profit. The matrix is formed by columns, obtaining the maximum of the column and subtracting from this value the payment of each alternative (Vitoriano, 2007). Company A will choose the row for which the greatest risk is minimum (*minimax*). In this way, said matrix will be such that its elements are given by:

$$\alpha_{ij} = \max a_{kj} - a_{ij}\,.$$

Company A will choose the row corresponding to:

$$\underset{j}{\text{MIN}}\,[\underset{i}{\text{MAX}}\,\alpha_{ij}] \quad \begin{cases} \forall i = 1, 2, ..., m, \\ \forall j = 1, 2, ..., n. \end{cases}$$

Thus, the matrix of damages obtained is as follows:

N

	E$_1$	0	1 940 718	238 867	0	0	1 940 718		M I
	E$_2$	850 925	0	238 867	0	0	850 925	←	N
A	E$_3$	2 552 776	1 701 851	0	0	0	2 552 776		I M
	E$_4$	4 254 627	3 403 702	1 701 851	1 462 984	0	4 254 627		A X

Consequently, the alternative E_2 (minimax) will be chosen.

5.6. Agrawal-Heady or Benefit Criterion

This criterion, like others, is based on the principle that, once the decision has been made, nature will always oppose it and, in view of this fact, the position of rejoicing in what is left to lose is adopted. A brief description of it will be made but without applying it in our example.

Here, it is worth turning the previous criterion of Savage[10], and instead of grieving for his partial error, the decision-maker can rejoice at his partial success. These partial successes of the decision maker are recorded in a new matrix: that of the amounts that company A stops losing due to not being completely wrong. With this criterion, ultimately, company A reaches a conclusion that departs from both Wald's and Savage's criteria.

---

[10] **Savage** was an American mathematician specialized in statistics. His best-known work dates from 1954, published the following year, and is titled *The Foundations of Statistics* (cited in the bibliography) in which he introduces certain elements on decision theory. In his work he mentions and elaborates subjectivity of the expected utility establishing the bases of Bayesian inference and its fruitful applications to the theory of strategy games.

In short, it should be emphasized that the decision-maker manifests himself as an absolute pessimist when applying the three criteria. What essentially distinguishes these criteria is the conformist, nonconformist or timorous mentality for which they were thought by their illustrious authors (Ballestero, 1973).

5.7. Expected value of perfect information (EVPI)

It is assumed that the knowledge about the states of nature could be modified. This modification can entail a cost, so it is worth asking what is the value of having this information?, or how much are you willing to pay for it? It is obvious that with more information the expected profit will also be greater (Vitoriano, 2007).

The *expected gain with perfect information* (EGPI) is defined to the mathematical expectation of the gain taking for each state of nature the best possible option. For the example at hand it would be, v. gr., in case a) above of section 4.3.:

$$EGPI = 3/5 \cdot 2{,}791{,}643 + 1/5 \cdot 1{,}940{,}718 + 1/15 \cdot 238{,}867 + 1/15 \cdot 0 + 1/15 \cdot 0 =$$

$$= 2{,}079{,}053.87 \text{ €}.$$

On the other hand, the *expected profit with uncertainty* (EPU) is estimated, that is, the expected profit with the decision made with any of the above criteria. Thus, if the selected decision is E_2, the EPU is € 1,940,718.00.

Finally, the *expected value of the perfect information* (EVPI) is defined as the difference between both previously defined gains, that is, in this case:

$$EVPI = EGPI - EPU = 2{,}079{,}053.87 - 1{,}940{,}718.00 = 138{,}335.87 \text{ €}.$$

Note that this is equivalent to using the Savage criterion (or the opportunity costs) with the minimum expected penalty (Vitoriano, 2007).

6. RESULT OF THE APPLIED CRITERIA

It can be seen, in short, that the application of the various criteria set forth may lead to the adoption of different strategies by company A.

This can be synthesized in the following summary table, with express indication of the 15 pure strategies chosen, according to each of the criteria adopted by the bidding company A.

<table>
<tr><td rowspan="15">MODALITY</td><td rowspan="9">WITH STATES OF NATURE</td><td>LAPLACE</td><td>E_2</td></tr>
<tr><td rowspan="2">EXPECTED GAIN</td><td>E_1</td></tr>
<tr><td>E_2</td></tr>
<tr><td>MOST LIKELY</td><td>E_1</td></tr>
<tr><td rowspan="2">MIDDLE SCENARIO</td><td>E_3</td></tr>
<tr><td>E_2</td></tr>
<tr><td rowspan="3">VARIABILITY OF RESULTS (β = 0.2)</td><td>E_2</td></tr>
<tr><td>E_1</td></tr>
<tr><td>E_3</td></tr>
<tr><td rowspan="6">NO STATES OF NATURE</td><td rowspan="3">NEUMANN OR WALD (PESSIMIST)</td><td>E_1</td></tr>
<tr><td>E_2</td></tr>
<tr><td>E_3</td></tr>
<tr><td>OPTIMISTIC</td><td>E_1</td></tr>
<tr><td>HURWICZ (with α = ½)</td><td>E_1</td></tr>
<tr><td>SAVAGE</td><td>E_2</td></tr>
</table>

Table 3. Summary of the results obtained.

It can be verified that among the 15 options calculated by application of all the criteria contemplated in this article, E_1 appears 6 times, E_2 6 times, E_3 3 times and E_4 not once, so it is concluded the desirability of company A deciding to carry out, in the escrow of the auction in question, a reduction taking into account the first 2 alternatives studied, that is, offering a decrease of 10-12%, or better still, making a weighted arithmetic mean of all options, like so:

$$R = \frac{6 \times 10\% + 6 \times 12\% + 3 \times 16\% + 0 \times 20\%}{15} = 12\% ,$$

therefore, the most rational determination to be adopted by company A should be to make a 12% drop in the auction on the tender budget for the corresponding work or service.

7. CONCLUSION

This exposition has tried to simplify the present problem as much as possible; the more data you had from the other competing companies, its complication would gradually increase although, in any case, the problem is solvable.

It should be borne in mind, on the other hand, that it is very convenient to apply these criteria when attending the auction. We can compare the case to that of a good betting player: handling his combinatorial theory, he concludes, for example, that he must fill in 644 columns to have any certainty of getting it right; and, nevertheless, the simple hobbyist, to have the same security, will probably need, without using any special statistical technique, to fill 2,000 or 3,000 columns, with the consequent cost difference.

Well, the case presented is very similar: if the probability of being awarded the work is the same, why do you have to drop by 16,561% if it does not matter if you do it by 12% from the probabilistic point of view? In addition, with this, 4.561% of the budget will be earned in all the certifications that are charged, which will entail, in the total of the work studied, an amount of:

$$\frac{4.561 \times 42\,546\,270.00}{100} = 1\,940\,535.38\ €\ ,$$

which represents, practically, 2 million euros, an amount that, of course, does not imply any negligible savings.

The determination of the losses in the auction, in short, is something that should not only be carried out with the exhaustive study of the technical project, the land and the financing conditions of the bidding company, but also through the application of reputable techniques of the Operational Research, such as Decision theory, which -as complicated as they may seem at first- immediately confer a high degree of profitability to the business management carried out.

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