# HOW MANY STUDENTS WERE EXAMINED?

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SUMMARY / ABSTRACT

The resolution of this problem, through the application of simple and well-known principles of the algebra of finite sets, reveals, in short, the applicability of these elementary techniques of modern algebra for the resolution of some real-life problems whose complexity it prevents them from an easy or immediate resolution at first sight.

**Key words**: exam, set, mathematical logic.

*RESUMEN*

*La resolución de este problema, mediante la aplicación de sencillos y conocidos principios del álgebra de los conjuntos finitos, pone de manifiesto, en definitiva, la aplicabilidad de estas técnicas elementales del álgebra moderna para la resolución de algunos problemas de la vida real cuya cierta complejidad les impide una fácil o inmediata resolución a primera vista.*

***Palabras clave****: examen, conjunto, lógica matemática.*

*RESUM*

*La resolució d'aquest problema, mitjançant l'aplicació de senzills i coneguts principis de l'àlgebra dels conjunts finits, posa de manifest, en definitiva, l'aplicabilitat d'aquestes tècniques elementals de l'àlgebra moderna per a la resolució d'alguns problemes de la vida real la complexitat dels quals els impedeix una fàcil o immediata resolució a primera vista.*

***Paraules clau****: examen, conjunt, lògica matemàtica.*

In a section of a certain newspaper of great circulation, in the heat of the summer heat, a curious *riddle or mathematical problem* appeared within the section corresponding to the "riddle of the week" that was posed, approximately, in the following terms:

“An exam consists of 2 different parts: 18 students passed the first, 23 students the second and 8 students both parts. Eleven students failed the 2 parts of the exam. How many students were tested? ”

Well, the solution that was given promptly the next day, in the same media, was expressed as follows:

“A total of 11 students failed both parts and 18 matched the first, so these two sets can be added together until they are **29**. That's already a good slice of the class. The problem is made more complicated by deciphering the rest. We know that 23 boys / girls responded well to the second part, but it turns out that we do not know if they did the previous part well or badly. How many of those who responded well to the first did the same in the second? According to the problem posed they were 8, so if we add 23 to the 29 that we already had clear we would be counting twice this small group of 8 students.

The **solution** is to subtract these 8 instead of adding them, therefore the correct answer is **44 students** (11 + 18 + 23 **-** 8 = 44)”.

The problem thus posed seems solved by the "old woman's account". And in fact it is so. If we intend to solve it, now, in a more orthodox or academic way from the point of view of mathematical science, we would have to embrace rudimentary concepts of Set Theory, and even pose the statement in another more attractive way that would, indeed, hinder , its resolution by the expeditious method previously used. In effect, the new statement of the problem could be the following:

“An exam consists of 2 different parts: 18 students passed the first, 23 the second and 8 both parts. A quarter of the students failed both parts of the exam. How many students were tested? ”

To achieve a better understanding of the exercise, we must consider its graphic representation using a simple Venn-Euler diagram like the one in the following figure, which will be remembered (even vaguely) by all those who have passed through secondary education. This is:

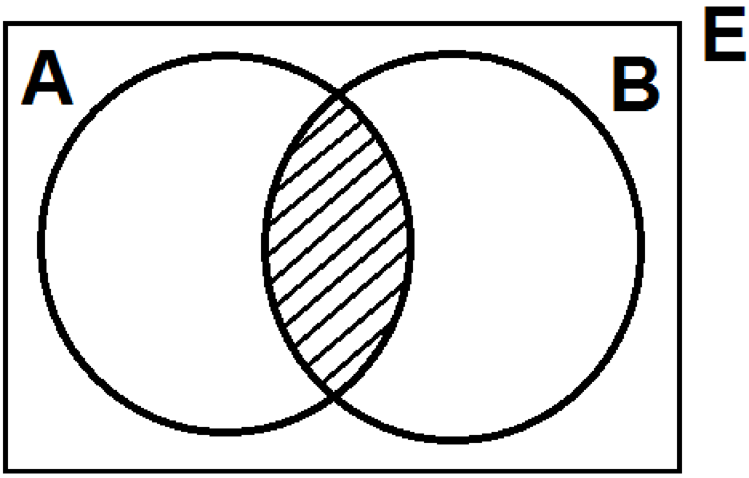


Fig.1. Venn-Euler diagram.

, being $\begin{cases} A \text{ (students who pass part 1)} = 18 \\ B \text{ (students who pass part 2)} = 23 \\ A \cap B \text{ (students passing both sides)} = 8 \\ \overline{A \cup B} \text{ (students who suspend everything)} = E/4 \end{cases}$

Obviously, we call "E" the universal set, whose number of elements (students), in short, is what we are looking for. For this, it is enough to consider the following equation that occurs in the case of two non-disjoint events or sets contemplated here (since there is an intersection between both finite sets of students):

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

In the proposal of the newspaper in question, a piece of information was offered that greatly simplified things, namely: $n(\overline{A \cup B}) = 11$ students (who suspended everything). This allows us to quickly deduce, but already in this case by orthodox procedures, the number of elements that make up the universal set, namely:

$$E = n(A \cup B) + 11 = n(A) + n(B) - n(A \cap B) + 11 = 18 + 23 - 8 + 11 = 44 \text{ students}$$

Now, with the new presentation of the problem advocated here, that data does not exist and we are only told that "a quarter of the students in the class fail the entire exam." With this, we will be left with the following equation that elegantly solves the total number of examinees:

$$E = n(A \cup B) + n(\overline{A \cup B}) = n(A) + n(B) - n(A \cap B) + 0.25 \cdot E,$$

and then, substituting the known values, we will have to:

$0.75 \cdot E = 18 + 23 - 8 = 33$, from where: $E = 33/0.75 =$ **44 students**, as it was intended to demonstrate.

Solving this problem, by applying simple and well-known principles of finite-set algebra, shows, in short, the applicability of these elementary techniques of modern algebra to the solution of certain real-life problems whose complexity they it prevents an easy or immediate resolution by other procedures that, colloquially, we could classify as “naked eye”.

The Set Theory, created by Cantor[1], is very useful in mathematics, as it is an important tool to study the relationships between a whole and its parts, while laying the groundwork to simplify definitions of concepts that were more complex, as in the case of the problem covered here.

Furthermore, set theory is rich enough to construct the rest of the objects and structures of interest in mathematics: numbers, functions, geometric figures, ...; and together with the mathematical logic it allows to study its foundations. Today, it is accepted that the set of axioms of the Zermelo-Fraenkel theory is sufficient to develop all of mathematics. In this discipline, it is common for cases of indemonstrable or contradictory properties to appear, such as the continuum hypothesis or the existence of an inaccessible cardinal. For this reason, the reasoning and techniques of set theory rely to a large extent on mathematical logic.

[1] Set theory and its basic foundations were developed by George Cantor (1845-1918), a German mathematician, towards the end of the 19th century. This theory tries to understand the properties of sets that are not related to the specific elements of which they are composed. Therefore, both theorems and axioms of set theory involve general sets, regardless of whether they contain physical objects or numbers. There are many practical applications of set theory such as the one developed in this exercise.